AGH University of Science and Technology
Faculty of Electrical Engineering, Automatics, Computer Science and Biomedical Engineering
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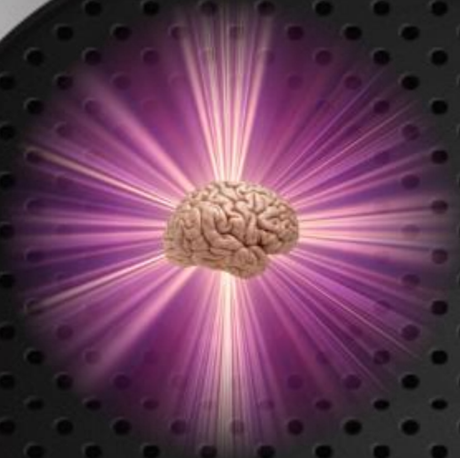

Computational Intelligence

Backpropagation and K-fold Cross Validation And Backpropagation Through Time

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Training Examples

How do we define a training dataset for supervised training?

Training Examples

for supervised training

Training examples are represented as a set of m pairs:

$$(X, Y) = \{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$$

where

m – is the number of examples

m_{train} – is the number of training examples

m_{test} – is the number of test examples

For vectorization, we stack the training examples in the matrix X as well as outputs Y:

$$X = \begin{bmatrix} x_1^{(1)} & \cdots & x_1^{(m)} \\ \vdots & \ddots & \vdots \\ x_{n_x}^{(1)} & \cdots & x_{n_x}^{(m)} \end{bmatrix} \in \mathbb{R}^{n_x \times m} \qquad Y = [y^{(1)} \quad \dots \quad y^{(m)}] \in \mathbb{R}^{1 \times m}$$

When we use the Python command to read or set the shape, the notation is:

$$X.shape = (n_x, m) \qquad Y.shape = (1, m)$$

Functions in Deep Learning

Logistic regression, loss functions and cost functions to set up the goal of training.

Logistic Regression

For the given x, we get the output prediction $\hat{y} = P(y = 1|x)$
where y is the desired output that will be trained using parameters:

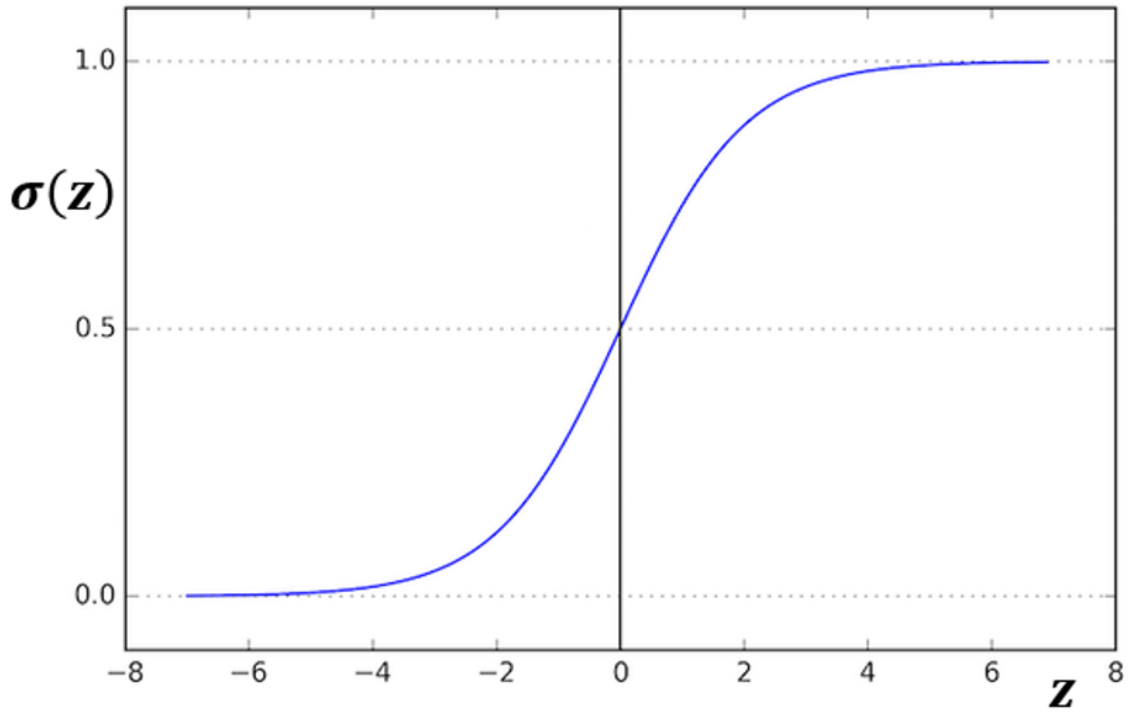
$w \in \mathbb{R}^{n_x}$

$b \in \mathbb{R}$

computing the output in the following way:

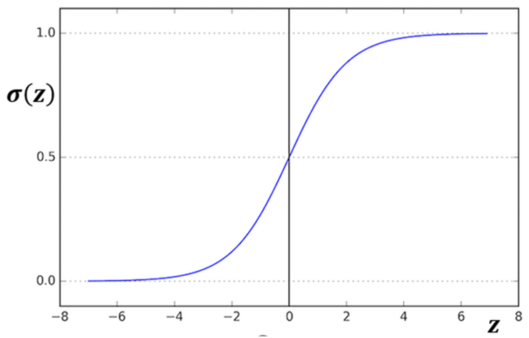
$$\hat{y} = \sigma\left(w^T x + b\right) = \sigma(z) = \frac{1}{1 + e^{-z}} \in (0, 1)$$

where σ is a sigmoid function:



Computing Sigmoid Function

We use numpy vectorization to compute sigmoid and its derivative for any input vector z:



$$\text{For } z \in \mathbb{R}^n,\ sigmoid(z) = sigmoid\begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{pmatrix} = \begin{pmatrix} \frac{1}{1+e^{-z_1}} \\ \frac{1}{1+e^{-z_2}} \\ \dots \\ \frac{1}{1+e^{-z_n}} \end{pmatrix} \quad (1)$$

$$sigmoid\_derivative(z) = \sigma'(z) = \sigma(z)(1-\sigma(z)) \quad (2)$$

```python
import numpy as np # this means you can access numpy functions by writing np.function() instead of numpy.function()

def sigmoid(z):
    a = 1 / (1 + np.exp(-z)) # Compute the sigmoid of z, where z can be a scalar or numpy array of any size
    return a

def sigmoid_derivative(z):
    a = sigmoid(z)         # Compute the gradient (slope, derivative) of the sigmoid function with respect to its input z.
    dJa = a * (1 - a)
    return dJa
```

```python
z = np.array([-2,-1,0,1, 2])
print ("sigmoid(z) = " + str(sigmoid(z)))
print ("sigmoid_derivative(z) = " + str(sigmoid_derivative(z)))
```

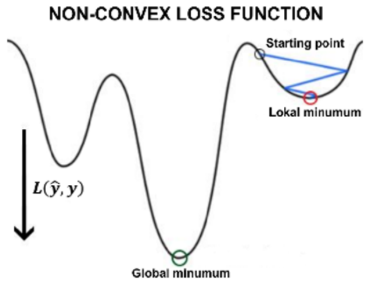
```
sigmoid(z) = [0.11920292 0.26894142 0.5        0.73105858 0.88079708]
sigmoid_derivative(z) = [0.10499359 0.19661193 0.25       0.19661193 0.10499359]
```

Logistic Regression Cost Function

We need to define logistic regression cost function to compute w and b parameters:

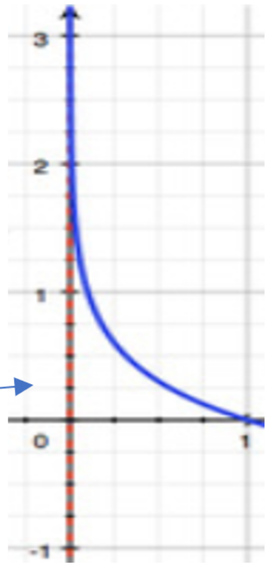
For the given training data set $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$ we want to get $\forall_i\ \hat{y}^{(i)} \approx y^{(i)}$

where $\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$ and $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}} \in (0, 1)$ (i) is the notation for i-th example



On this basis, we can define a loss function, called also an error function, for a single example that measures how good the output $\hat{y}$ is when the desired (trained) label is y:

The absolute error function $L_1(\hat{y}, y) = |\hat{y} - y|$ or the squared error function: $L_2(\hat{y}, y) = (\hat{y} - y)^2$ might seem like a good choice for this measure, but today we do not usually do this in this way because the optimization problem for it becomes not convex, so the gradient descent algorithm cannot find the global optimum of such loss functions easily!



We need to define the loss function in such a way that the function will be convex, so we use:

$$L_3(\hat{y}, y) = -(y\ log\ \hat{y} + (1 - y) log(1 - \hat{y}))$$

Consider two bounding cases:

If $y = 0$ then $L(\hat{y}, y) = -\ log\ (1 - \hat{y})$, so to minimize it, $log\ (1 - \hat{y})$ must be large and $\hat{y}$ small ($\hat{y} \rightarrow 0$).

If $y = 1$ then $L(\hat{y}, y) = -\ log\ \hat{y}$, so to minimize it, $log\ \hat{y}$ and $\hat{y}$ must be large ($\hat{y} \rightarrow 1$).

Finally, we define a cost function that measures the error on the entire training data set (for all examples):

$$J(w, b) = \frac{1}{m} \sum_{i=1}^{m} L(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} \left(y^{(i)}\ log\ \hat{y}^{(i)} + (1 - y^{(i)}) log(1 - \hat{y}^{(i)})\right)$$

Popular Loss Functions

The loss functions are used to evaluate the performance of the models.

The bigger our loss is, the more different our predictions ($\hat{y}$) are from the true values (y). In deep learning, we use optimization algorithms like Gradient Descent to train models and minimize the cost.

L1 loss is defined as an absolute distance between vectors $\hat{y}$ and y of the size n:

$$L_1(\hat{y}, y) = \sum_{j=0}^{n} |y_j - \hat{y}_j| \tag{1}$$

L2 loss is defined as a square distance between vectors $\hat{y}$ and y of the size n:

$$L_2(\hat{y}, y) = \sum_{j=0}^{n} (y_j - \hat{y}_j)^2 \tag{2}$$

L2 loss is defined between vectors $\hat{y}$ and y of the size n in the following way:

$$L_3(\hat{y}, y) = -\sum_{j=0}^{n} (y_j log(\hat{y}_j) + (1 - y_j)(1 - log(\hat{y}_j))) \tag{3}$$

```python
def L1(yhat, y):
    loss1 = np.sum(np.abs(y-yhat))
    return loss1

def L2(yhat, y):
    loss2 = np.sum(np.dot(y-yhat,y-yhat))
    return loss2

def L3(yhat, y):
    loss3 = - np.sum(y * np.log(yhat) + (1-y) * np.log(1-yhat))
    return loss3
```

```python
yhat = np.array([.78, .89, .12, .08, .97])
y = np.array([1, 1, 0, 0, 1])
print("Loss1 = " + str(L1(yhat,y)))
print("Loos2 = " + str(L2(yhat,y)))
print("Loos3 = " + str(L3(yhat,y)))
```

```
Loss1 = 0.5599999999999999
Loos2 = 0.0822
Loos3 = 0.6066693634880955
```

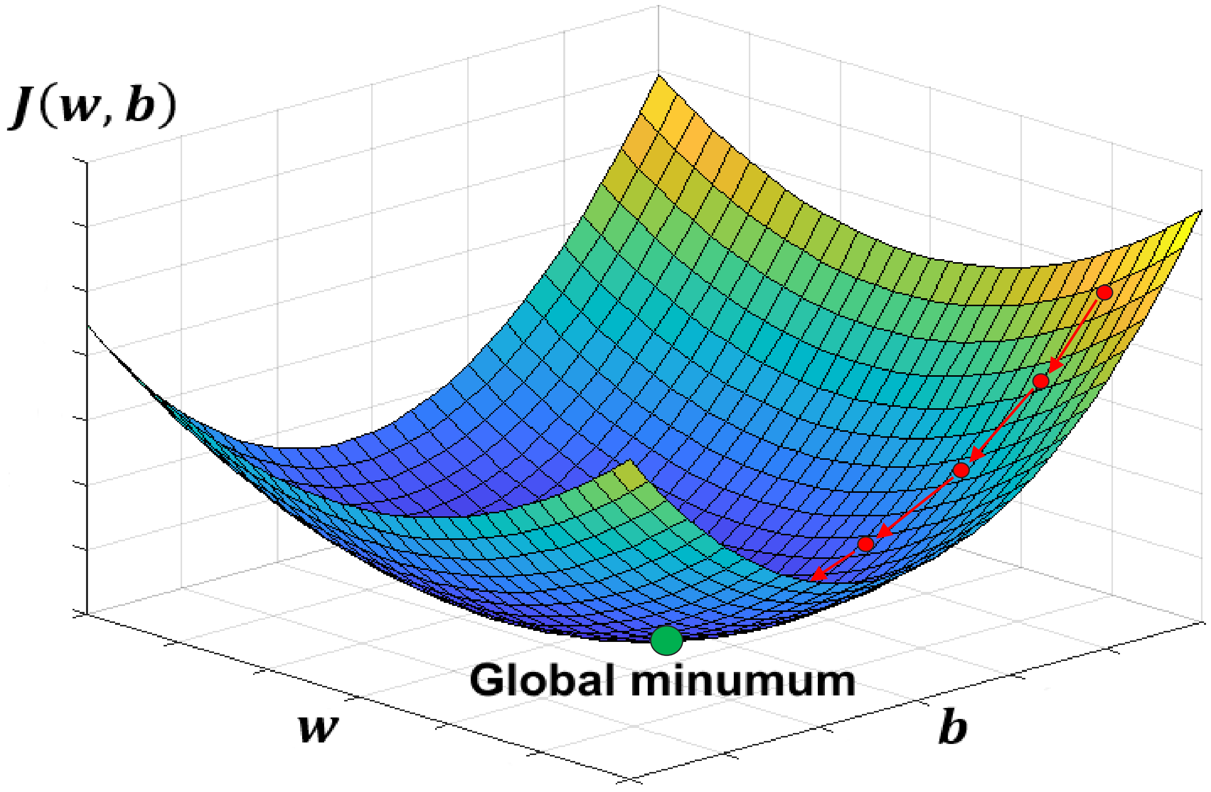
Gradient Descent Algorithm

We need to derivate activation functions to use gradient descent training algorithm.

Gradient Descent

We have to minimize the cost function *J* for a given training data set to achieve as correct prediction for input data as possible:

$$J(w,b)=\frac{1}{m}\sum_{i=1}^{m}L\left(\hat{y}^{(i)},y^{(i)}\right)=-\frac{1}{m}\sum_{i=1}^{m}\left(y^{(i)}\,log\,\hat{y}^{(i)}+\left(1-y^{(i)}\right)log\left(1-\hat{y}^{(i)}\right)\right)$$



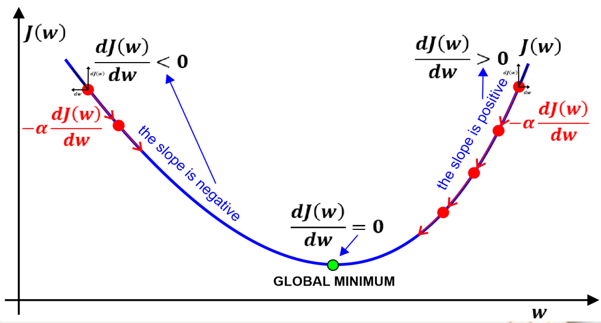
Here, *w* is 1D, but its dimension is bigger in real.

To minimize the cost function we calculate partial derivatives where $\frac{dJ(w,b)}{dw}$ and $\frac{dJ(w,b)}{db}$ of J with respect to parameters w and b and repeatedly use them to update them with a step α – called a learning rate:

$$w := w-\alpha\frac{dJ(w,b)}{dw}$$

$$b := b-\alpha\frac{dJ(w,b)}{db}$$

Partial derivatives $\frac{dJ(w,b)}{dw}=\frac{\partial J(w,b)}{\partial w}$ and $\frac{dJ(w,b)}{db}=\frac{\partial J(w,b)}{\partial b}$ represent the slopes of the J function:



Calculus of the Gradient Descent

Basic Derivatives Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

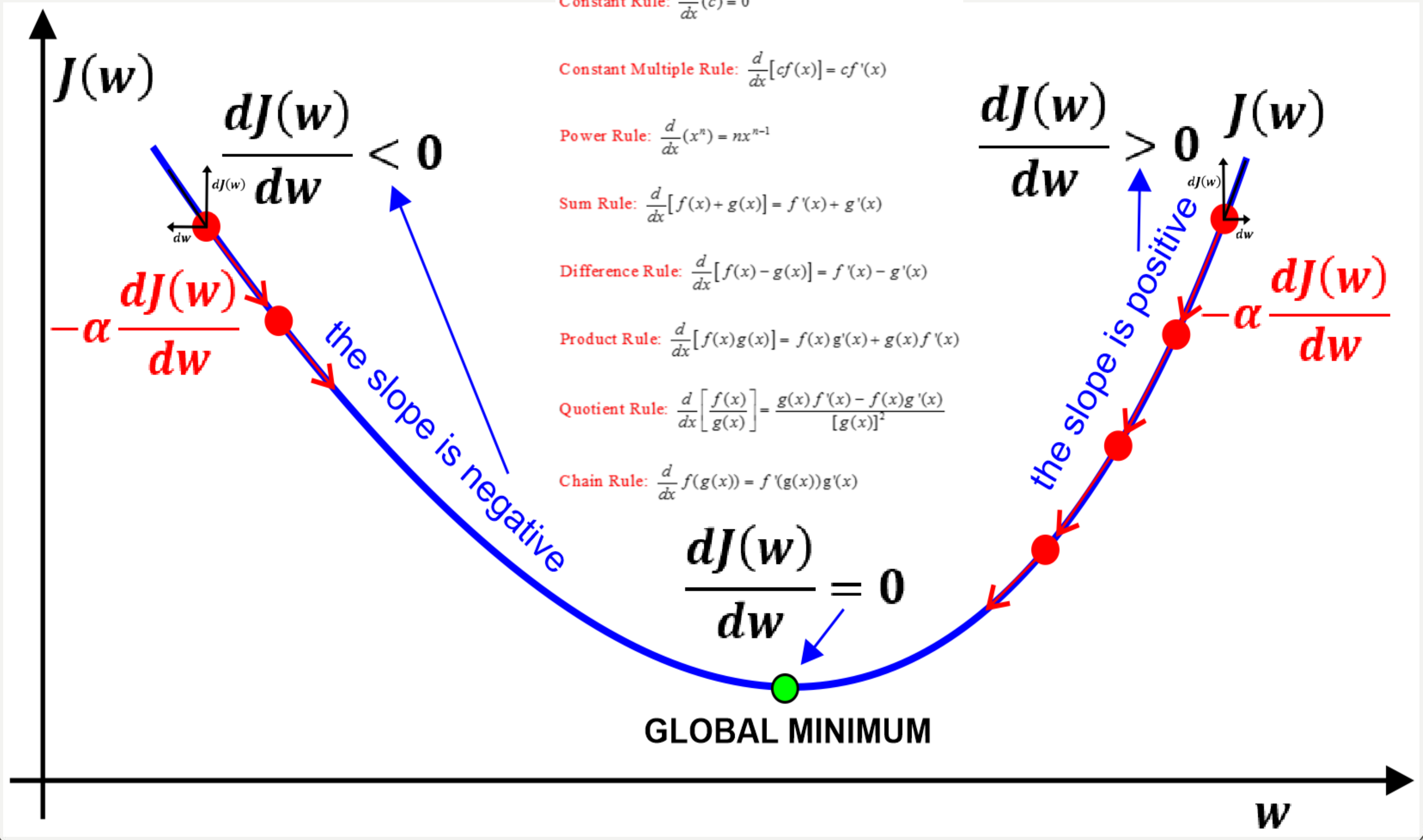
Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$



The main idea of the Gradient Descent algorithm is to go in the reverse direction to the gradient (the descent slope):

Derivative Rules

The Gradient Descent algorithm uses partial derivatives calculated after the following rules:

Basic Derivatives Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

Difference Rule: $\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative Rules

Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$
$$\frac{d}{dx}(a^x) = a^x \ln a$$
$$\frac{d}{dx}(e^{g(x)}) = e^{g(x)} g'(x)$$
$$\frac{d}{dx}(a^{g(x)}) = \ln(a)\, a^{g(x)} g'(x)$$

Logarithmic Functions

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$$
$$\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$$
$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, x > 0$$
$$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x) \ln a}$$

Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Inverse Trigonometric Functions

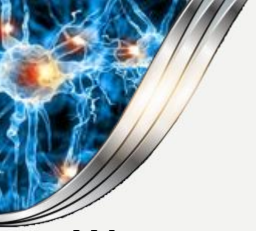
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$$
$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}, x \neq \pm 1$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}, x \neq \pm 1, 0$$
$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}, x \neq \pm 1, 0$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$
$$\frac{d}{dx}(\cosh x) = \sinh x$$
$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$
$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$
$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$
$$\frac{d}{dx}(\coth x) = -\operatorname{csch} x$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$
$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, x > 1$$
$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, |x| < 1$$
$$\frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{|x|\sqrt{1-x^2}}, x \neq 0$$
$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}, 0 < x < 1$$
$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}, |x| > 1$$



Gradient Descent for Logistic Regression

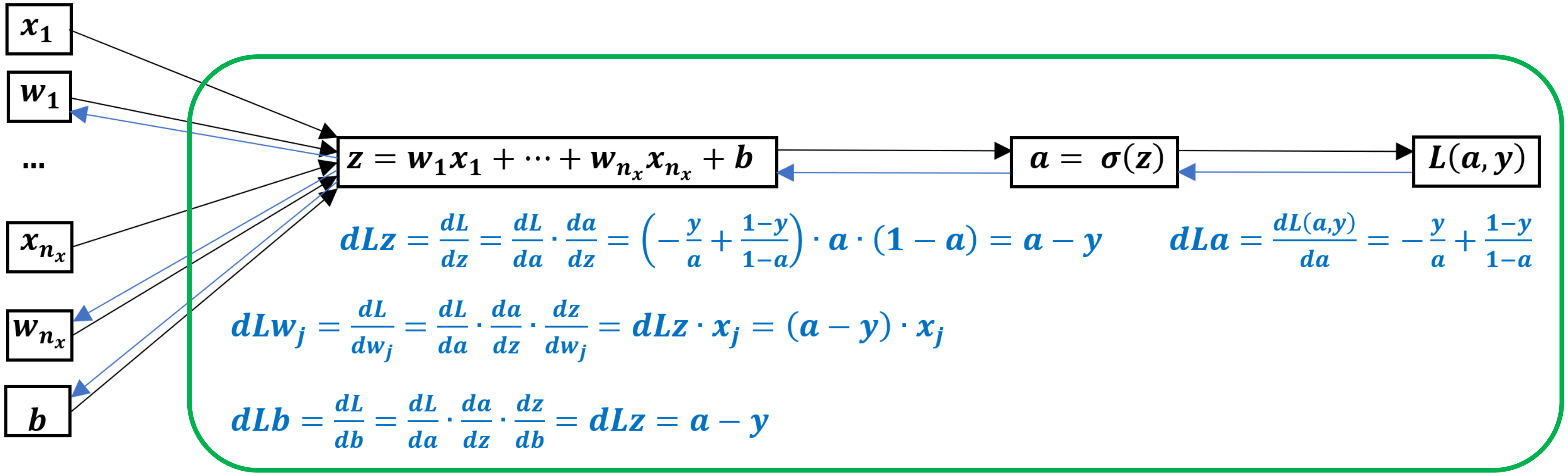
We use a computational graph for the presentation of forward and backward operations for a single neuron implementing logistic regression for the weighted sum of inputs *x*:

Use a computational graph to present operations of computation of the logistic regression and its derivatives:

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$L(a, y) = -\big(y \log a + (1 - y)\log(1 - a)\big)$$



$$dLz = \frac{dL}{dz} = \frac{dL}{da} \cdot \frac{da}{dz} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right) \cdot a \cdot (1 - a) = a - y \qquad dLa = \frac{dL(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$dLw_j = \frac{dL}{dw_j} = \frac{dL}{da} \cdot \frac{da}{dz} \cdot \frac{dz}{dw_j} = dLz \cdot x_j = (a - y) \cdot x_j$$

$$dLb = \frac{dL}{db} = \frac{dL}{da} \cdot \frac{da}{dz} \cdot \frac{dz}{db} = dLz = a - y$$

Finally, we get the update-rules for the logistic regression using the gradient descent algorithm:

$$w_j := w_j - \alpha \cdot dLw_j = w_j - \alpha \cdot (a - y) \cdot x_j$$

$$b := b - \alpha \cdot dLb = b - \alpha \cdot (a - y)$$

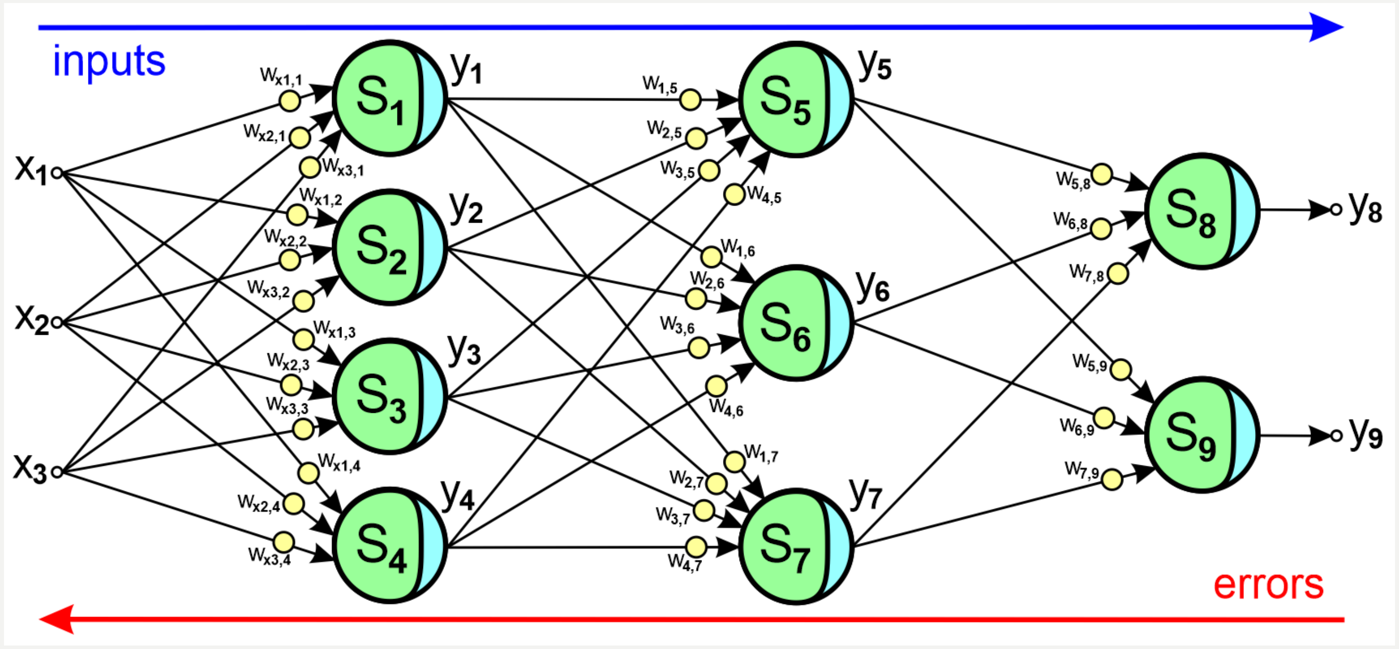
Backpropagation Algorithm

How artificial neural networks are trained?

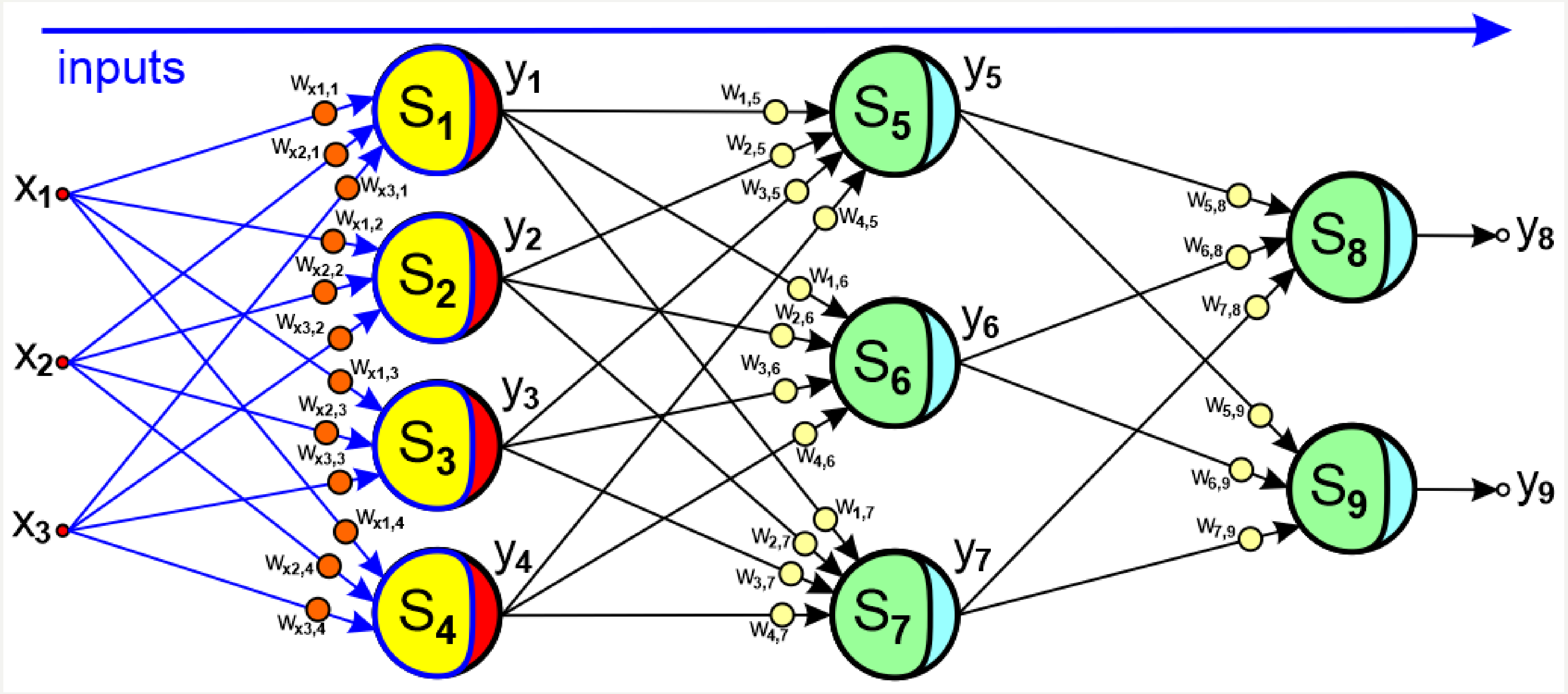
Network Training Process

Last layers of neural networks for classification or regression tasks are constructed from dense layers which can be trained using the most popular backpropagation algorithm which includes two main phases:

1. **The input propagation phase propagates the inputs throughout all hidden layers to the output layer neurons. In this phase, neurons calculate weighted sums of inputs taken from the neurons in the previous layer or the input of the network (x_1, …, x_3).**
2. **The error propagation phase propagates back the errors (delta values) computed on the outputs of the neural network. In this phase, neurons calculate weighted sums of errors (delta values) taken from the neurons of the next layer.**



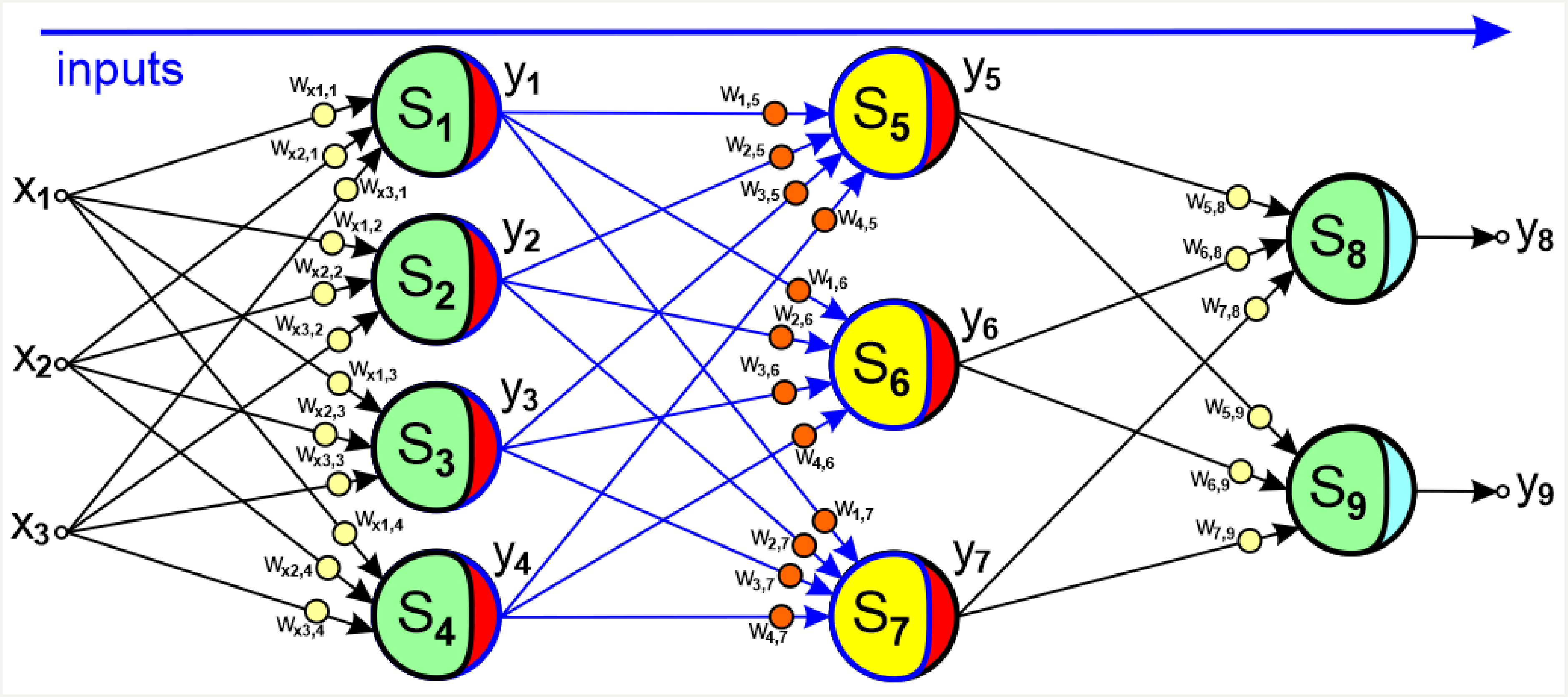
Backpropagation Algorithm



First, the inputs x_1, x_2, x_3 stimulate neurons in the first hidden layer.
The neurons compute weighted sums S_1, S_2, S_3, S_4, and output values y_1, y_2, y_3, y_4 that become inputs for the neurons of the next hidden layer:

$$S_n = \sum_{k=1}^{3} x_k \cdot w_{x_k,n} \qquad y_n = f(S_n)$$

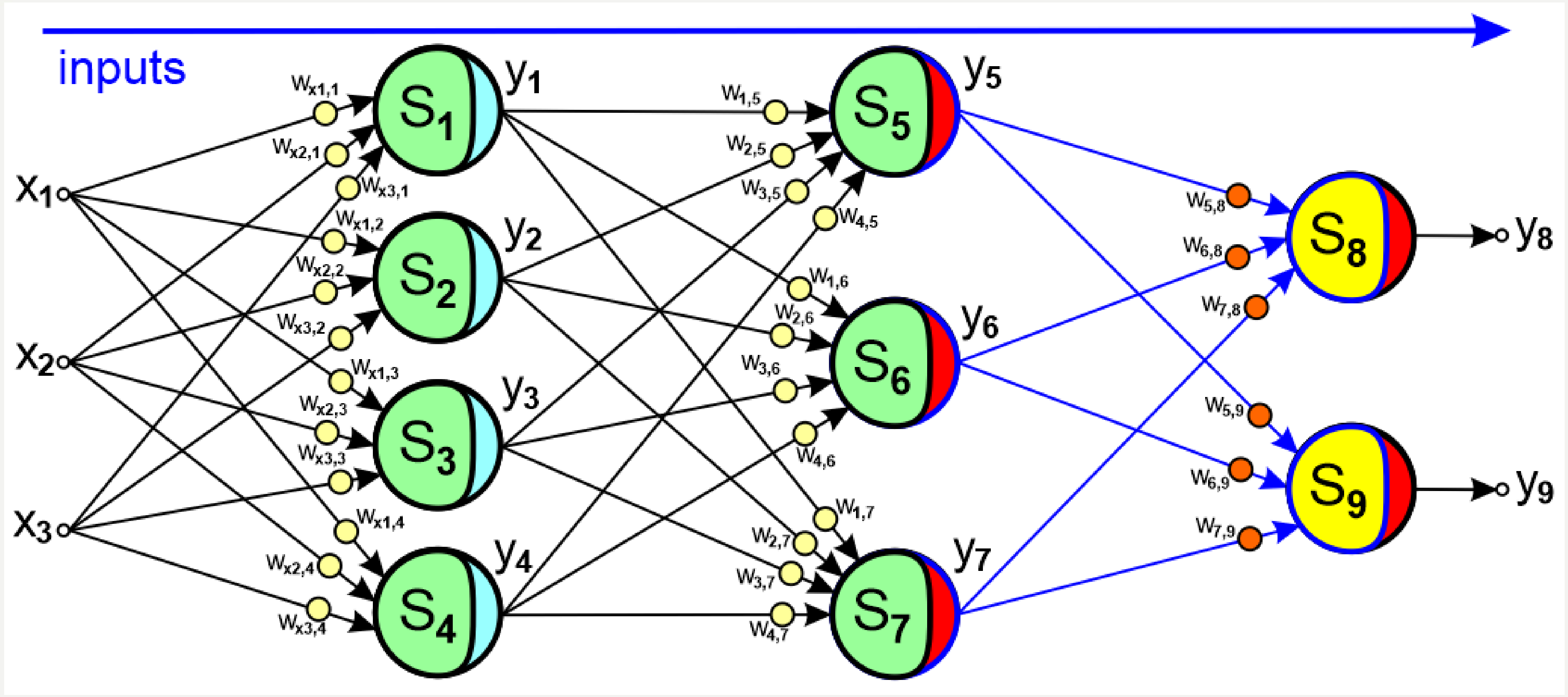
Backpropagation Algorithm



Second, the outputs y_1, y_2, y_3 ,y_4 stimulate neurons in the second hidden layer. The neurons compute weighted sums S_5, S_6, S_7, and output values y_5, y_6, y_7 that become inputs for the neurons of the output layer:

$$S_n = \sum_{k=1}^{4} y_k \cdot w_{k,n} \qquad y_n = f(S_n)$$

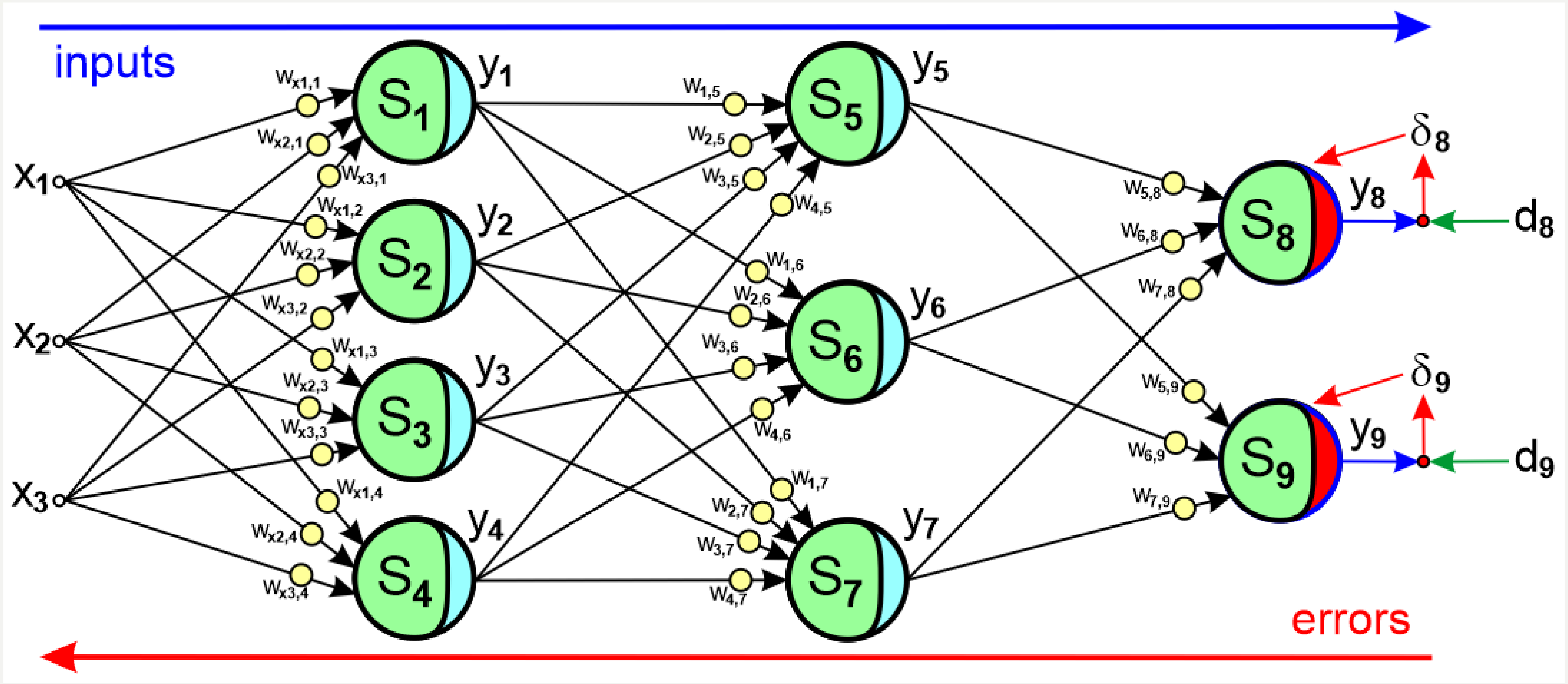
Backpropagation Algorithm



Finally, the outputs y_5, y_6, y_7 stimulate neurons in the output layer. The neurons compute weighted sums S_8 and S_9, and output values y_8, y_9 that are the outputs of the neural network as well:

$$S_n = \sum_{k=5}^{7} y_k \cdot w_{k,n} \qquad y_n = f(S_n)$$

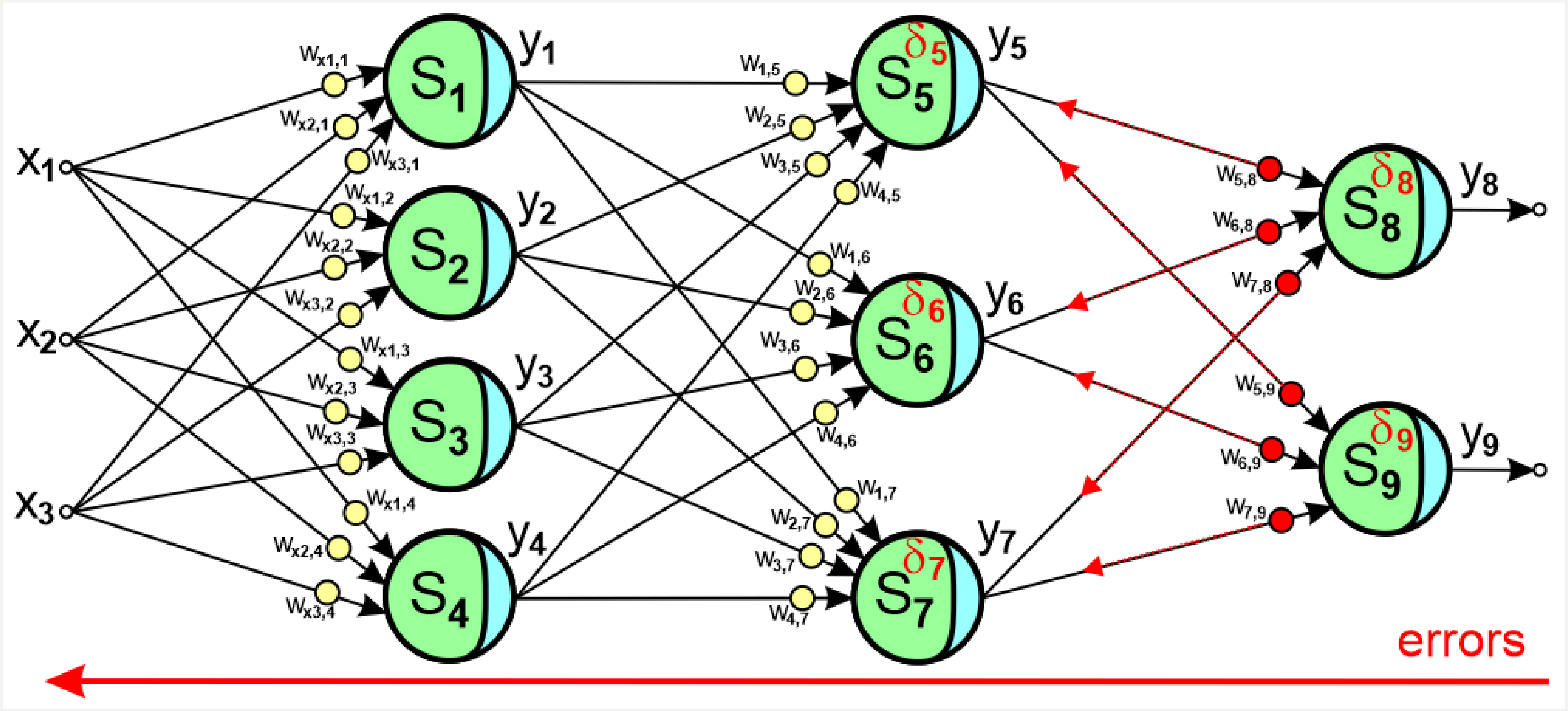
Backpropagation Algorithm



Next, the outputs y_8, y_9 are compared with the desired outputs d_8, d_9 and the errors δ_8, δ_9 are computed. These errors will be propagated back in order to compute corrections of weights from the connected inputs neurons.

$$\boldsymbol{\delta_n = d_n - y_n}$$

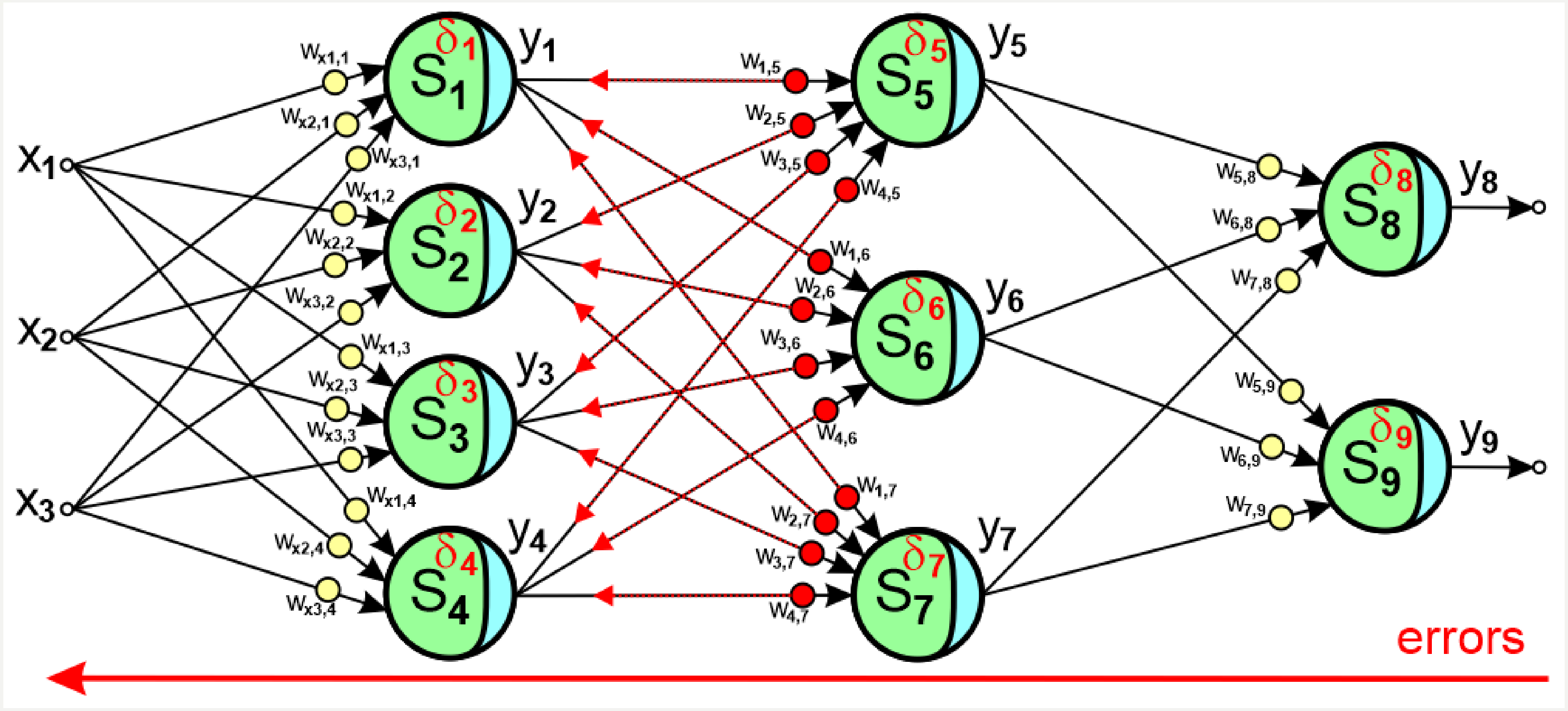
Backpropagation Algorithm



The errors δ_8 and δ_9 are used for corrections of the weights of the inputs connections y_5, y_6, y_7, and propagated back along the input connections to the neurons of the previous layer in order to compute their errors δ_5, δ_6, δ_7:

$$\Delta \mathbf{w}_{k,n} = -\eta \cdot \delta_n \cdot (1 - y_n) \cdot y_n \cdot y_k \qquad \delta_k = \sum_{n=8}^{9} \delta_n \cdot \mathbf{w}_{k,n} \cdot (1 - y_n) \cdot y_n$$

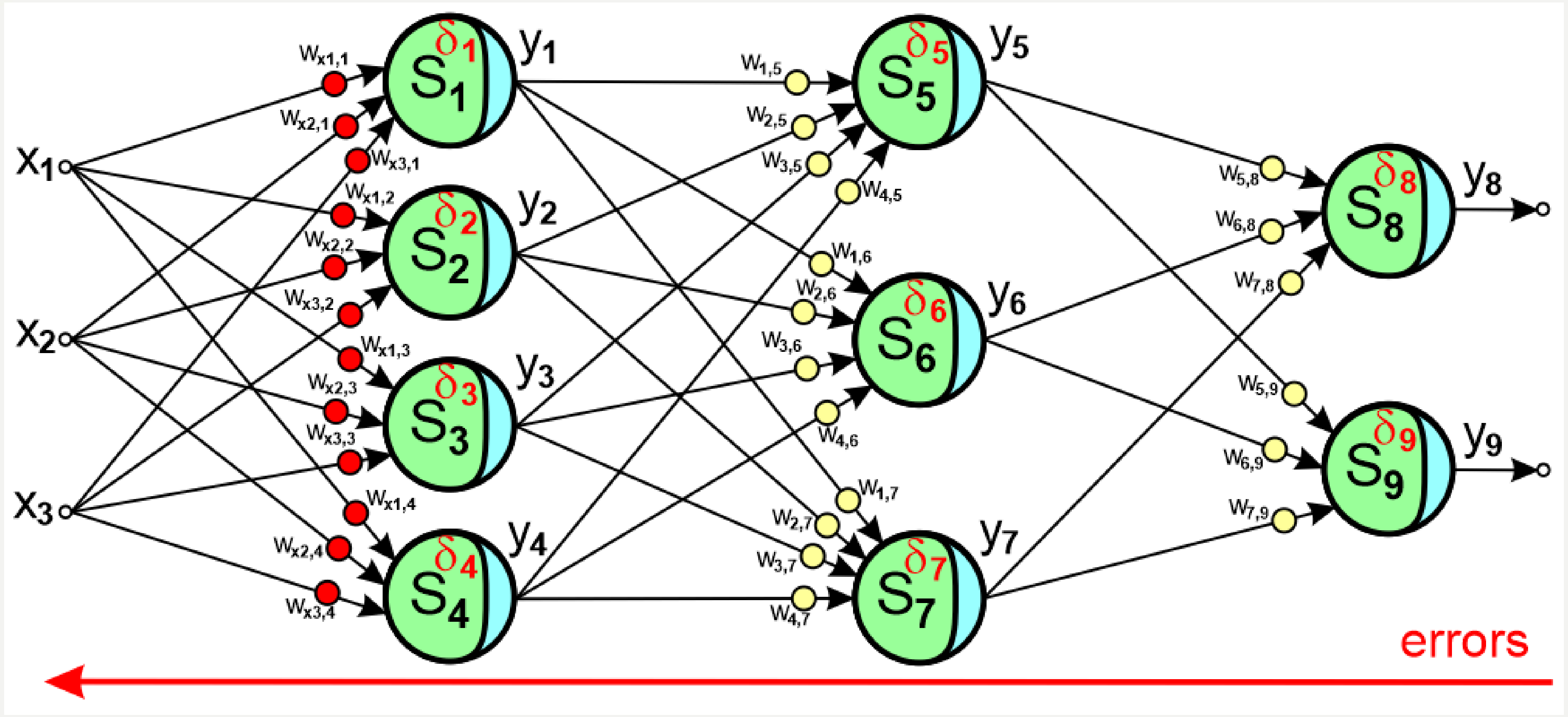
Backpropagation Algorithm



Next, the errors δ_5, δ_6, and δ_7 are used for corrections of the weights of the inputs connections y_1, y_2, y_3, y_4, and propagated back along the input connections to the neurons of the previous layer in order to compute their errors δ_1, δ_2, δ_3, δ_4:

$$\Delta \mathbf{w}_{k,n} = -\eta \cdot \delta_n \cdot (1 - y_n) \cdot y_n \cdot y_k \qquad \delta_k = \sum_{n=5}^{7} \delta_n \cdot \mathbf{w}_{k,n} \cdot (1 - y_n) \cdot y_n$$

Backpropagation Algorithm



Finally, the errors δ_1, δ_2, δ_3, δ_4 are used for corrections of the weights of the inputs x_1, x_2, x_3:

$$\Delta \mathbf{w}_{k,n} = -\boldsymbol{\eta} \cdot \boldsymbol{\delta}_n \cdot (1 - \boldsymbol{y}_n) \cdot \boldsymbol{y}_n \cdot \boldsymbol{y}_k$$

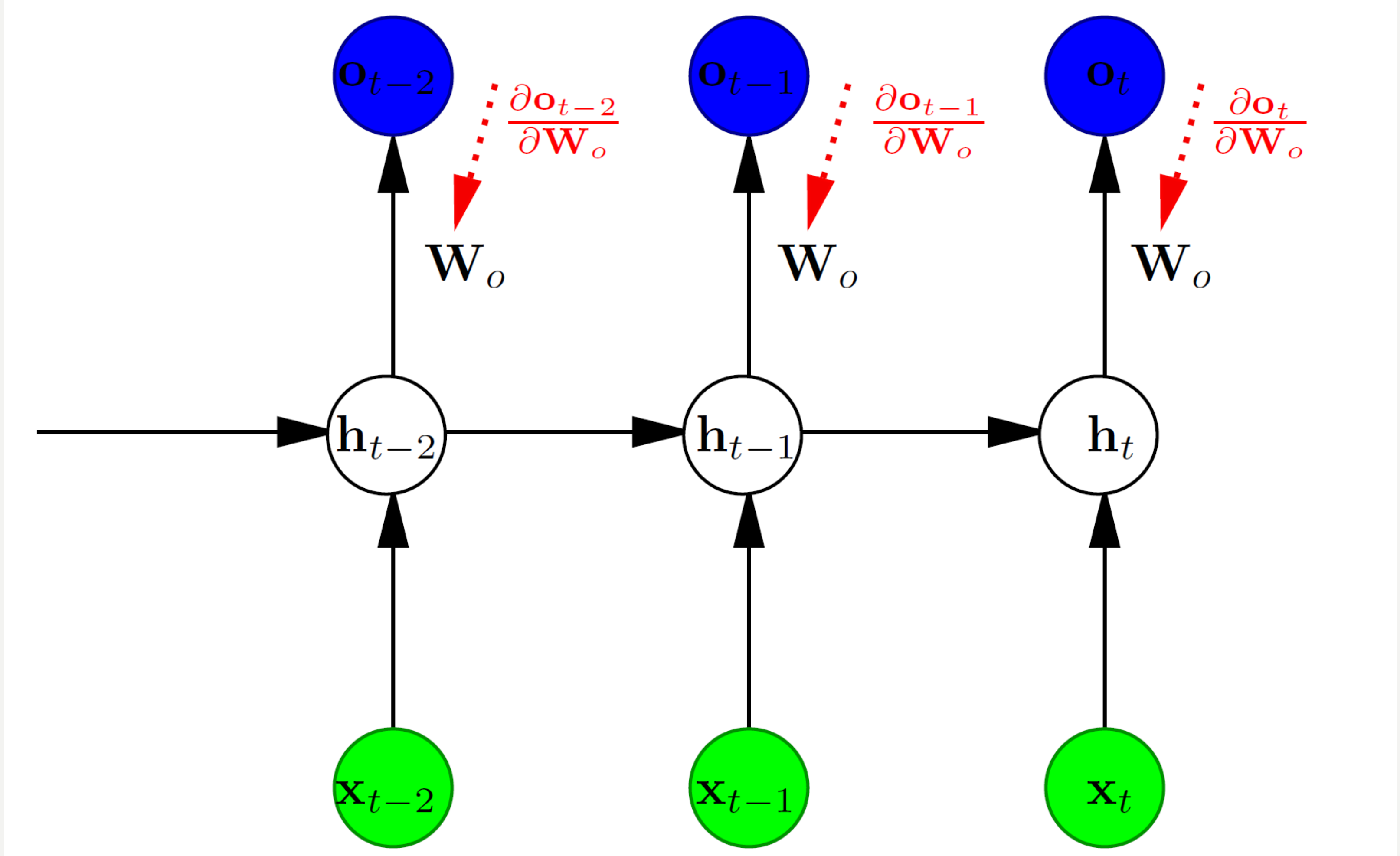
Back-Propagation Through Time

How to use backpropagation

in Recurrent Neural Networks?

Back-Propagation Through Time (BPTT)

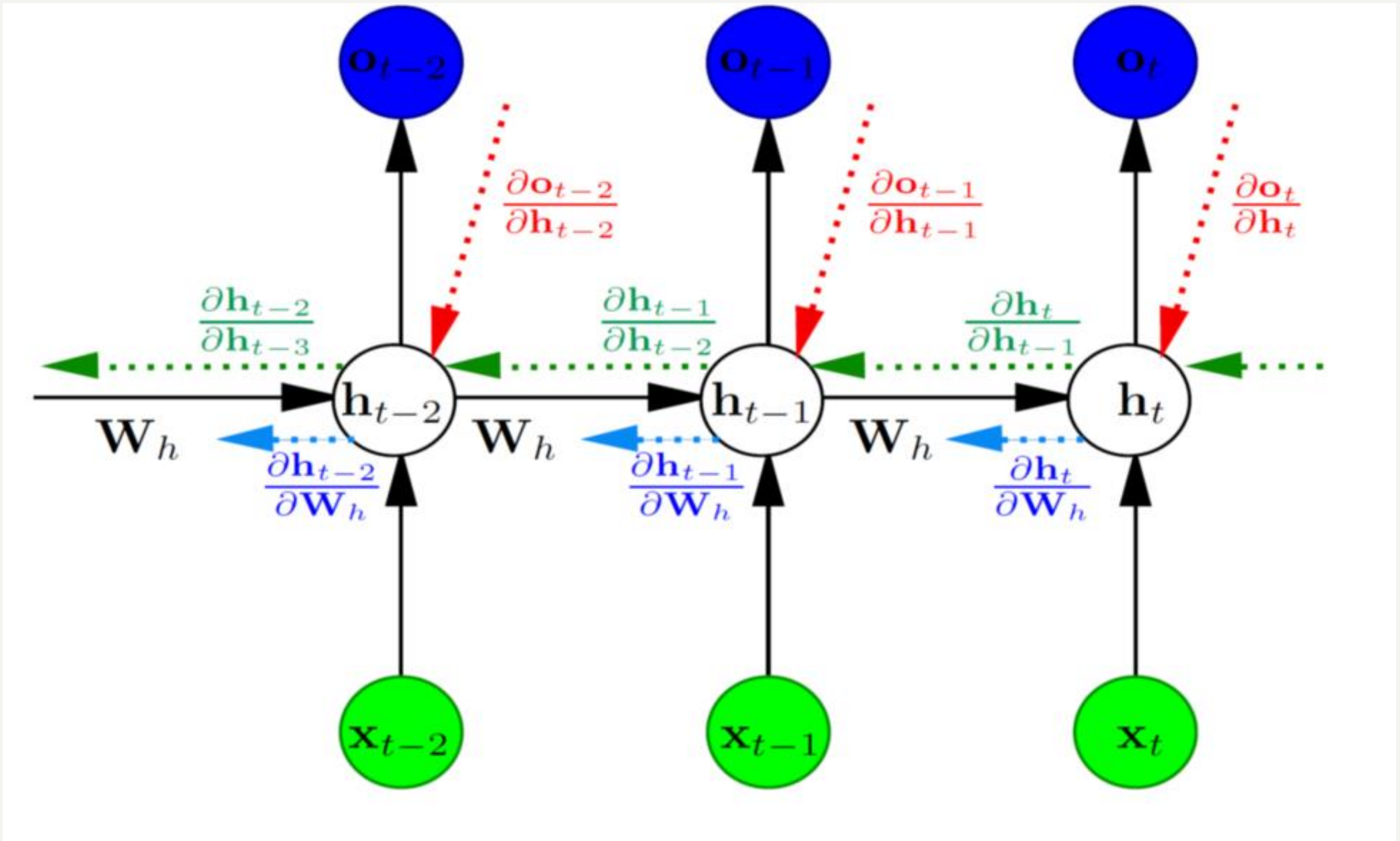
The backpropagation algorithm can be adapted to RNNs:



$$\frac{\partial \mathbf{E}}{\partial \mathbf{W}_o} = \cdots + \frac{\partial \mathbf{E}_{t-2}}{\partial \mathbf{o}_{t-2}} \frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{W}_o} + \frac{\partial \mathbf{E}_{t-1}}{\partial \mathbf{o}_{t-1}} \frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{W}_o} + \frac{\partial \mathbf{E}_t}{\partial \mathbf{o}_t} \frac{\partial \mathbf{o}_t}{\partial \mathbf{W}_o}$$

Back-Propagation Through Time (BPTT)

The backpropagation algorithm can be adapted to RNNs:



$\mathbf{o}_{t-2}$
$\mathbf{o}_{t-1}$
$\mathbf{o}_{t}$
$\frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{h}_{t-1}}$
$\frac{\partial \mathbf{o}_{t}}{\partial \mathbf{h}_{t}}$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{h}_{t-3}}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{h}_{t-1}}$
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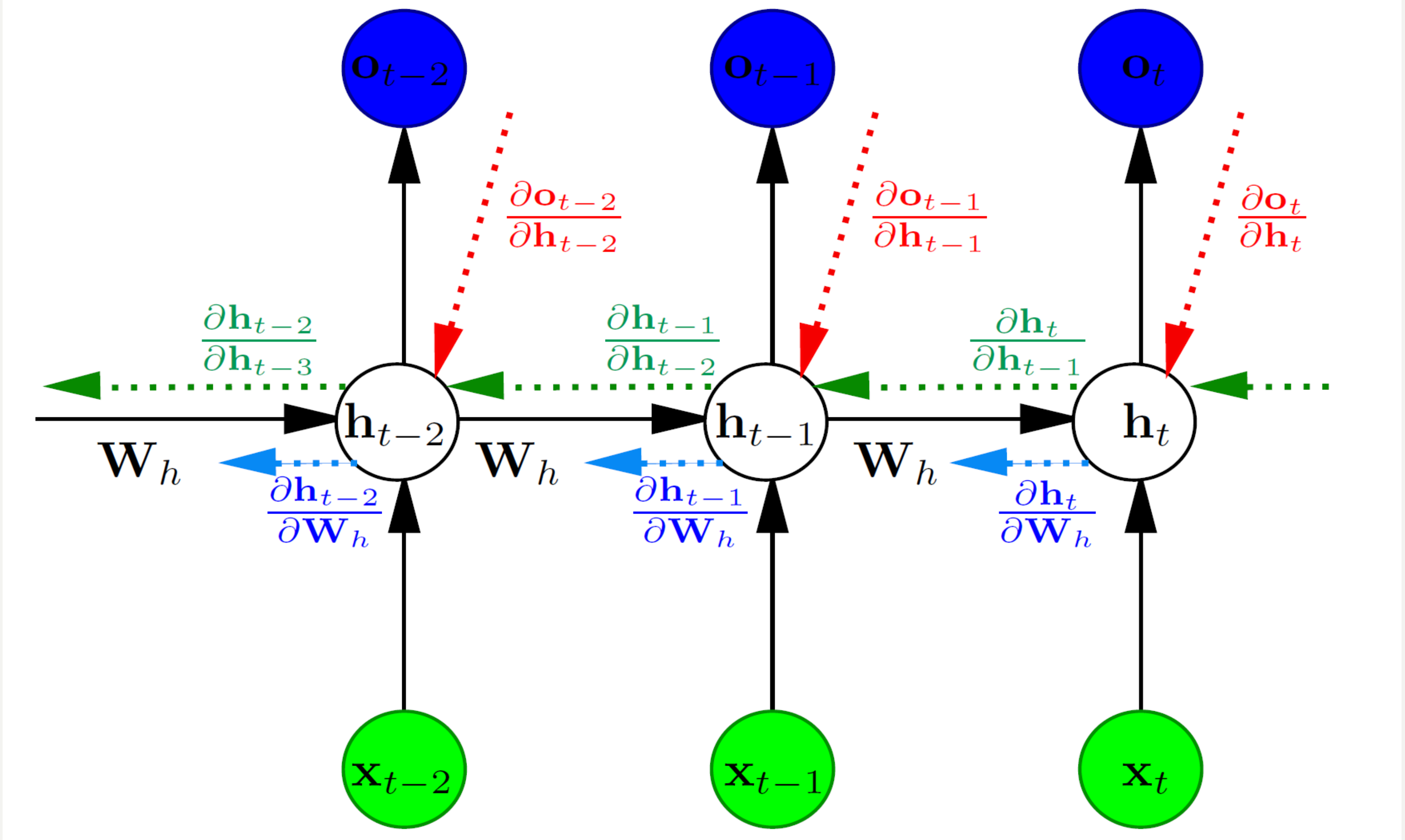
$\mathbf{h}_{t-1}$
$\mathbf{h}_{t}$
$\mathbf{W}_h$
$\mathbf{W}_h$
$\mathbf{W}_h$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{W}_{h}}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{W}_{h}}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{W}_{h}}$
$\mathbf{x}_{t-2}$
$\mathbf{x}_{t-1}$
$\mathbf{x}_{t}$

$$\frac{\partial \mathbf{o}_t}{\partial \mathbf{W}_h} = \sum_{t'=1}^{t} \frac{\partial \mathbf{o}_t}{\partial \mathbf{h}_t} \frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t'}} \frac{\partial \mathbf{h}_{t'}}{\partial \mathbf{W}_h}, \text{ where } \frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t'}} = \prod_{j=t'+1}^{t} \frac{\partial \mathbf{h}_j}{\partial \mathbf{h}_{j-1}}$$

Back-Propagation Through Time (BPTT)

The backpropagation algorithm can be adapted to RNNs:

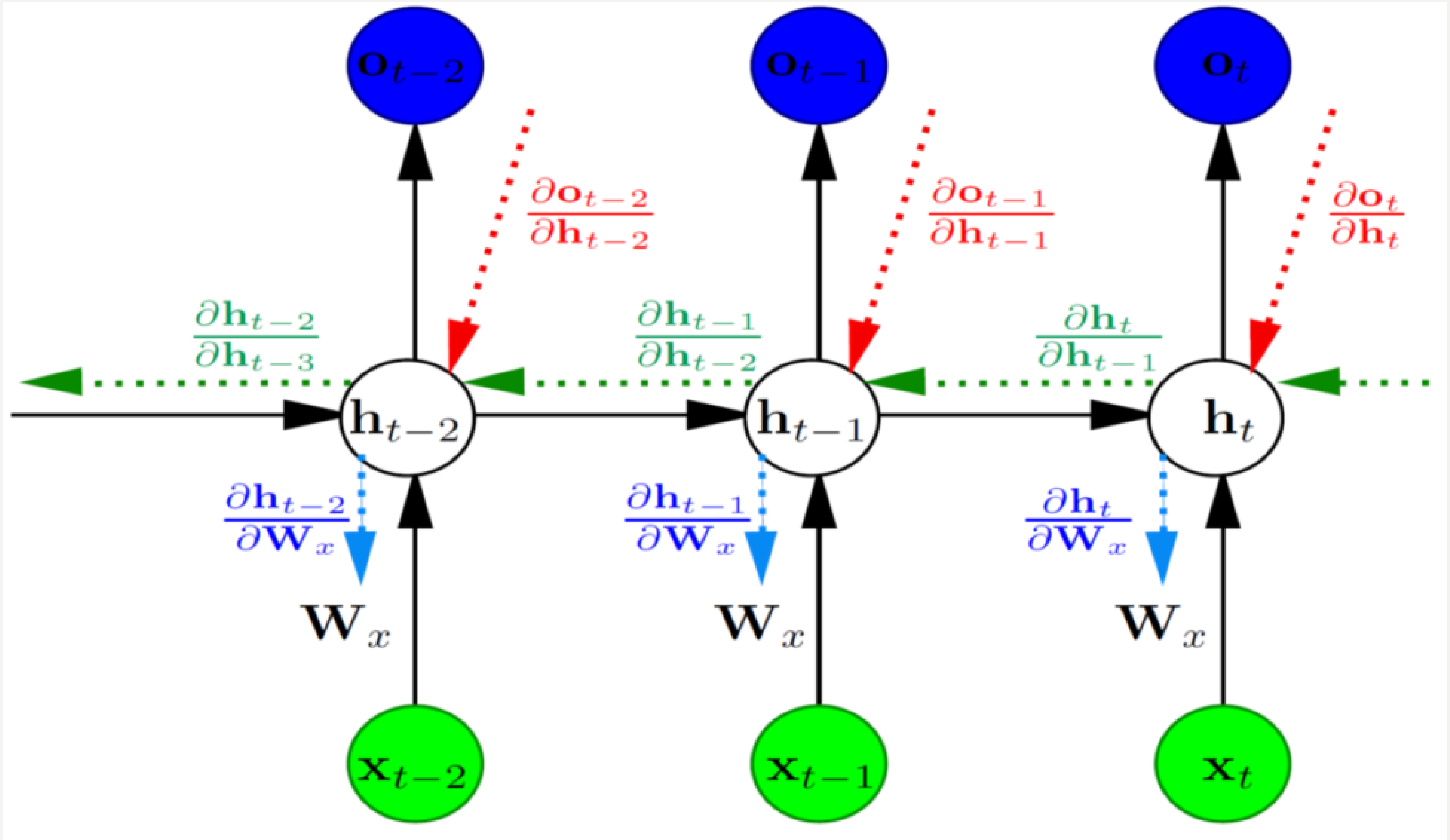


$\mathbf{o}_{t-2}$
$\mathbf{o}_{t-1}$
$\mathbf{o}_{t}$
$\frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{h}_{t-1}}$
$\frac{\partial \mathbf{o}_{t}}{\partial \mathbf{h}_{t}}$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{h}_{t-3}}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{h}_{t-1}}$
$\mathbf{h}_{t-2}$
$\mathbf{h}_{t-1}$
$\mathbf{h}_{t}$
$\mathbf{W}_h$
$\mathbf{W}_h$
$\mathbf{W}_h$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{W}_h}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{W}_h}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{W}_h}$
$\mathbf{x}_{t-2}$
$\mathbf{x}_{t-1}$
$\mathbf{x}_{t}$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{W}_h} = \cdots + \frac{\partial \mathbf{E}_{t-2}}{\partial \mathbf{o}_{t-2}} \frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{W}_h} + \frac{\partial \mathbf{E}_{t-1}}{\partial \mathbf{o}_{t-1}} \frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{W}_h} + \frac{\partial \mathbf{E}_{t}}{\partial \mathbf{o}_{t}} \frac{\partial \mathbf{o}_{t}}{\partial \mathbf{W}_h}$$

Back-Propagation Through Time (BPTT)

The backpropagation algorithm can be adapted to RNNs:

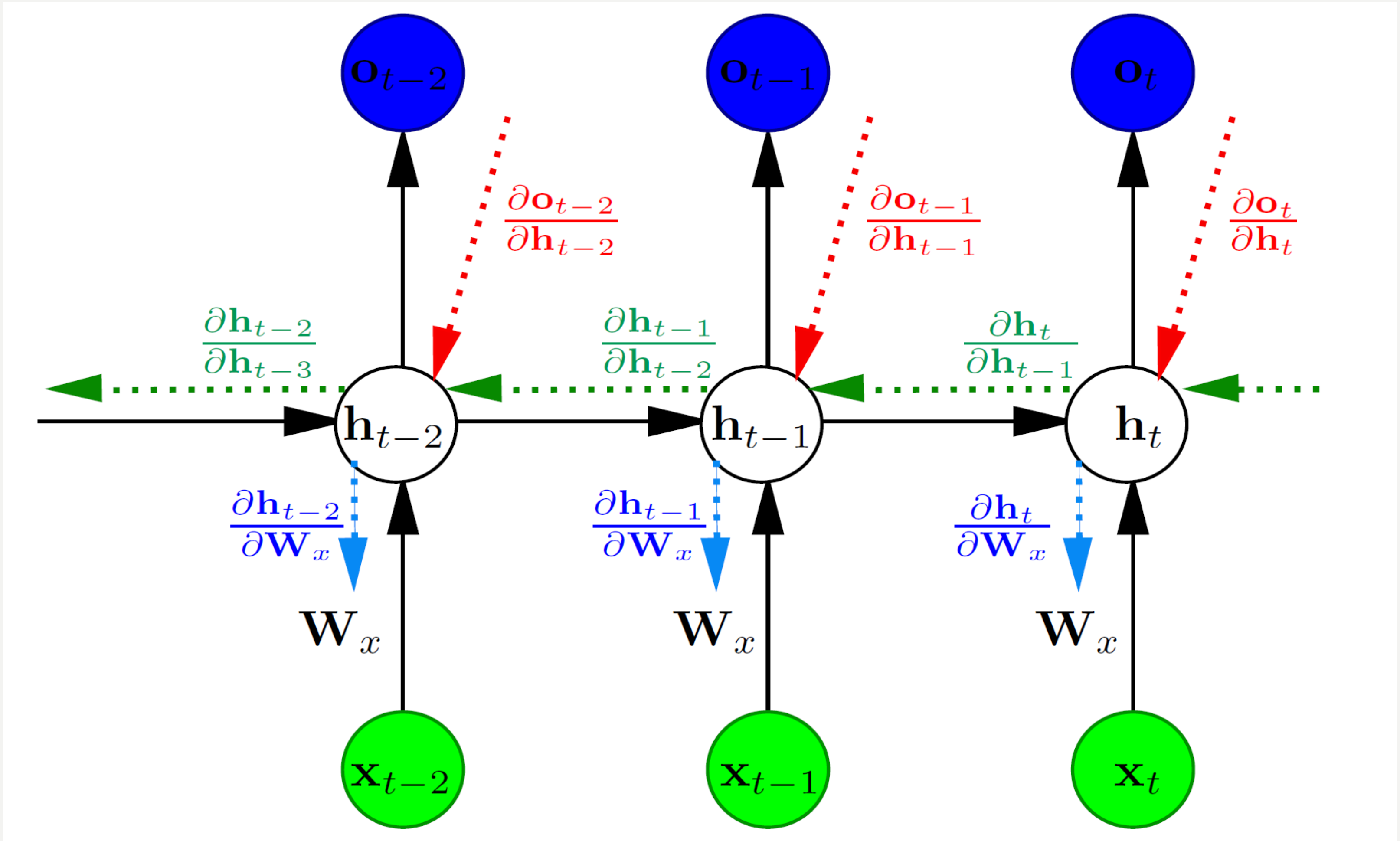


$\mathbf{o}_{t-2}$
$\mathbf{o}_{t-1}$
$\mathbf{o}_t$
$\frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{h}_{t-1}}$
$\frac{\partial \mathbf{o}_{t}}{\partial \mathbf{h}_{t}}$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{h}_{t-3}}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{h}_{t-1}}$
$\mathbf{h}_{t-2}$
$\mathbf{h}_{t-1}$
$\mathbf{h}_t$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{W}_x}$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{x}_{t-2}$
$\mathbf{x}_{t-1}$
$\mathbf{x}_t$

$$\frac{\partial \mathbf{o}_t}{\partial \mathbf{W}_x} = \sum_{t'=1}^{t} \frac{\partial \mathbf{o}_t}{\partial \mathbf{h}_t} \frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t'}} \frac{\partial \mathbf{h}_{t'}}{\partial \mathbf{W}_x}$$

Back-Propagation Through Time (BPTT)

The backpropagation algorithm can be adapted to RNNs:

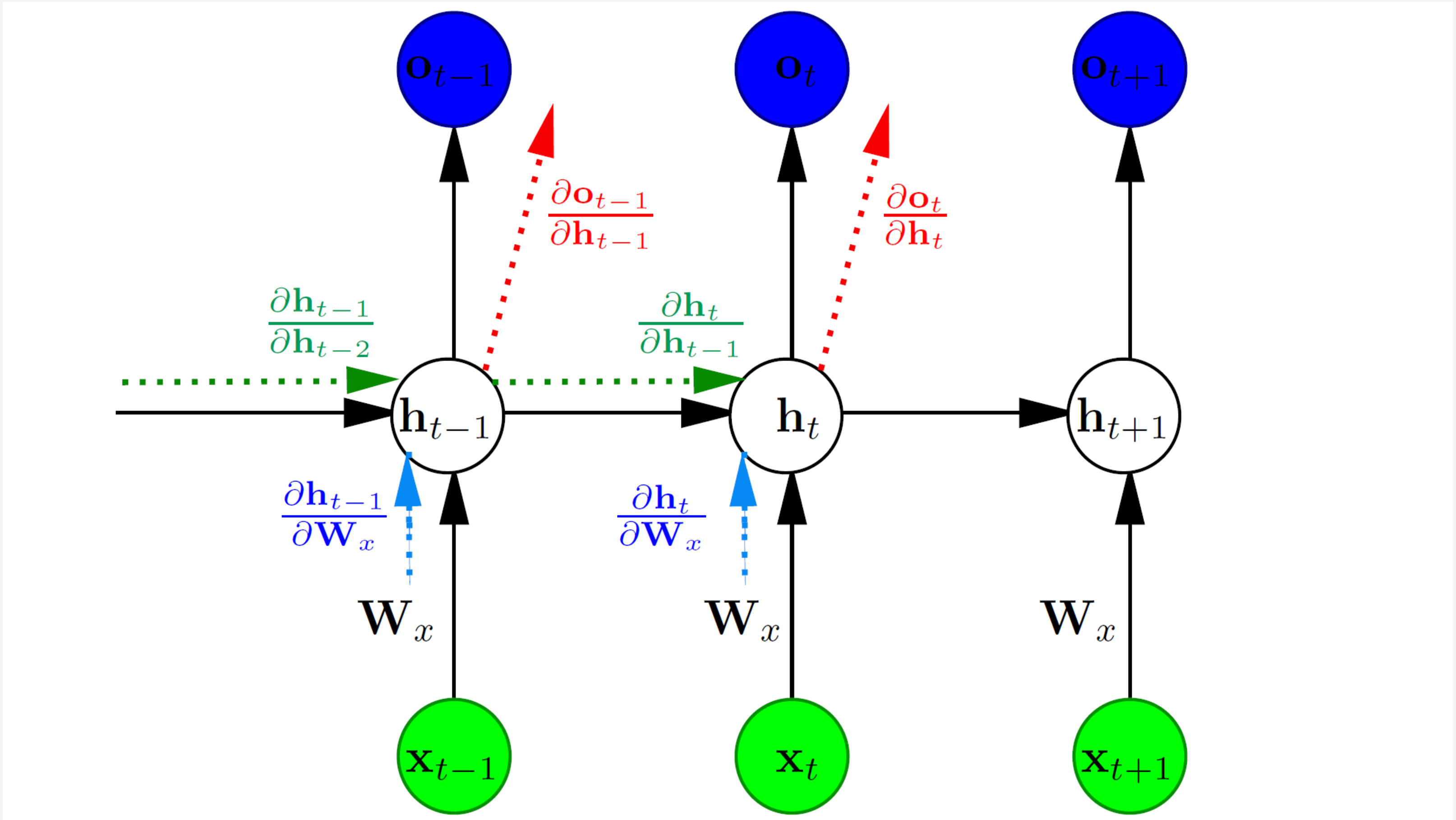


$\mathbf{o}_{t-2}$
$\mathbf{o}_{t-1}$
$\mathbf{o}_t$
$\frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{h}_{t-1}}$
$\frac{\partial \mathbf{o}_{t}}{\partial \mathbf{h}_{t}}$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{h}_{t-3}}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{h}_{t-1}}$
$\mathbf{h}_{t-2}$
$\mathbf{h}_{t-1}$
$\mathbf{h}_t$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_{t}}{\partial \mathbf{W}_x}$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{x}_{t-2}$
$\mathbf{x}_{t-1}$
$\mathbf{x}_t$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{W}_x} = \cdots + \frac{\partial \mathbf{E}_{t-2}}{\partial \mathbf{o}_{t-2}} \frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{W}_x} + \frac{\partial \mathbf{E}_{t-1}}{\partial \mathbf{o}_{t-1}} \frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{W}_x} + \frac{\partial \mathbf{E}_{t}}{\partial \mathbf{o}_{t}} \frac{\partial \mathbf{o}_{t}}{\partial \mathbf{W}_x}$$

Real-Time Recurrent Learning (RTRL)

Real-Time Recurrent Learning (RTRL) computes partial derivatives during the forward phase:

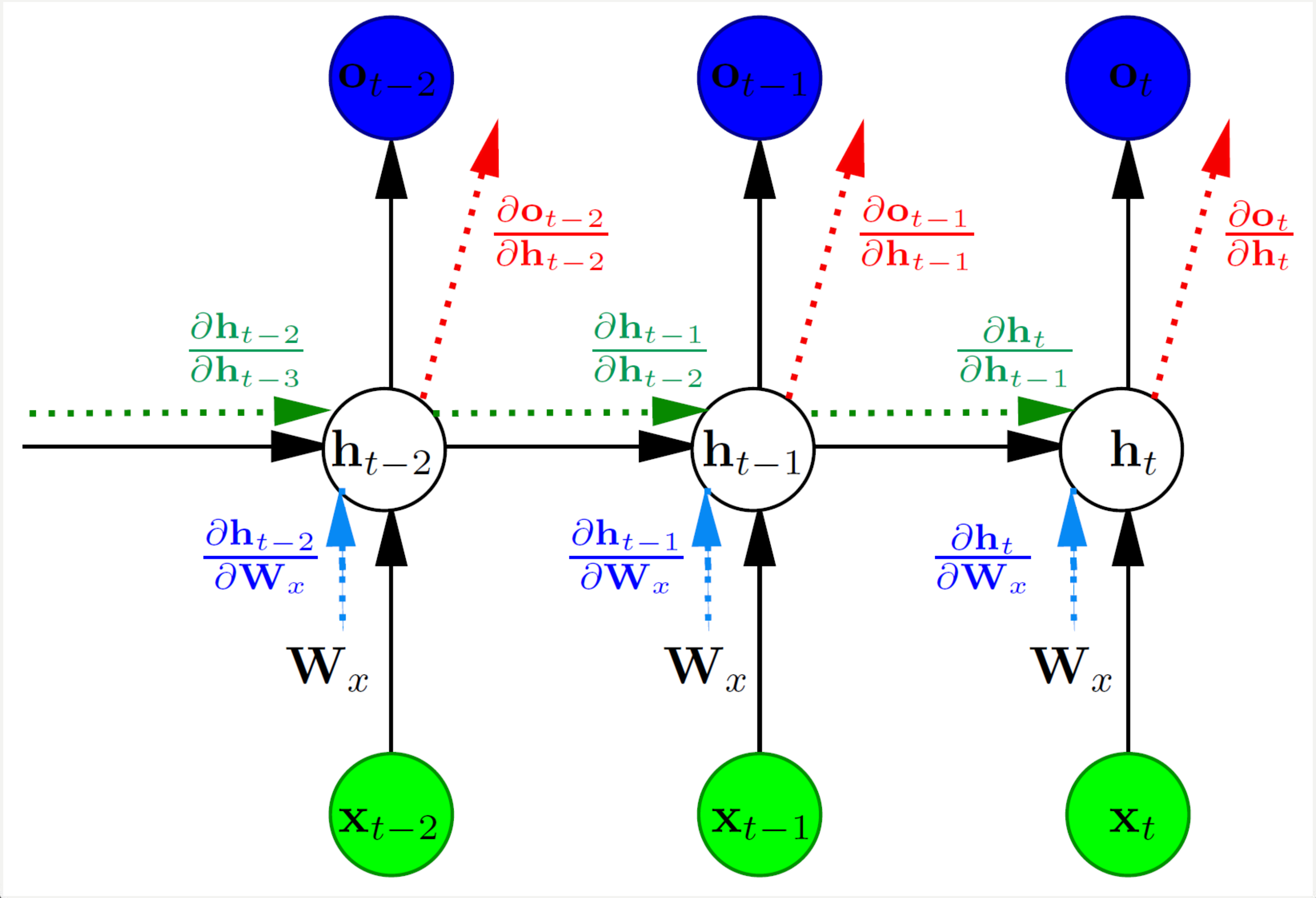


$\mathbf{o}_{t-1}$
$\mathbf{o}_t$
$\mathbf{o}_{t+1}$
$\frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{h}_{t-1}}$
$\frac{\partial \mathbf{o}_t}{\partial \mathbf{h}_t}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t-1}}$
$\mathbf{h}_{t-1}$
$\mathbf{h}_t$
$\mathbf{h}_{t+1}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_t}{\partial \mathbf{W}_x}$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{x}_{t-1}$
$\mathbf{x}_t$
$\mathbf{x}_{t+1}$

$$\frac{\partial \mathbf{E}|_t}{\partial \mathbf{W}_x} = \cdots + \frac{\partial \mathbf{E}_{t-1}}{\partial \mathbf{o}_{t-1}} \frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{W}_x} + \frac{\partial \mathbf{E}_t}{\partial \mathbf{o}_t} \frac{\partial \mathbf{o}_t}{\partial \mathbf{W}_x} = \frac{\partial \mathbf{E}|_{t-1}}{\partial \mathbf{W}_x} + \frac{\partial \mathbf{E}_t}{\partial \mathbf{o}_t} \frac{\partial \mathbf{o}_t}{\partial \mathbf{W}_x}$$

Real-Time Recurrent Learning (RTRL)

Real-Time Recurrent Learning (RTRL) computes partial derivatives during the forward phase:

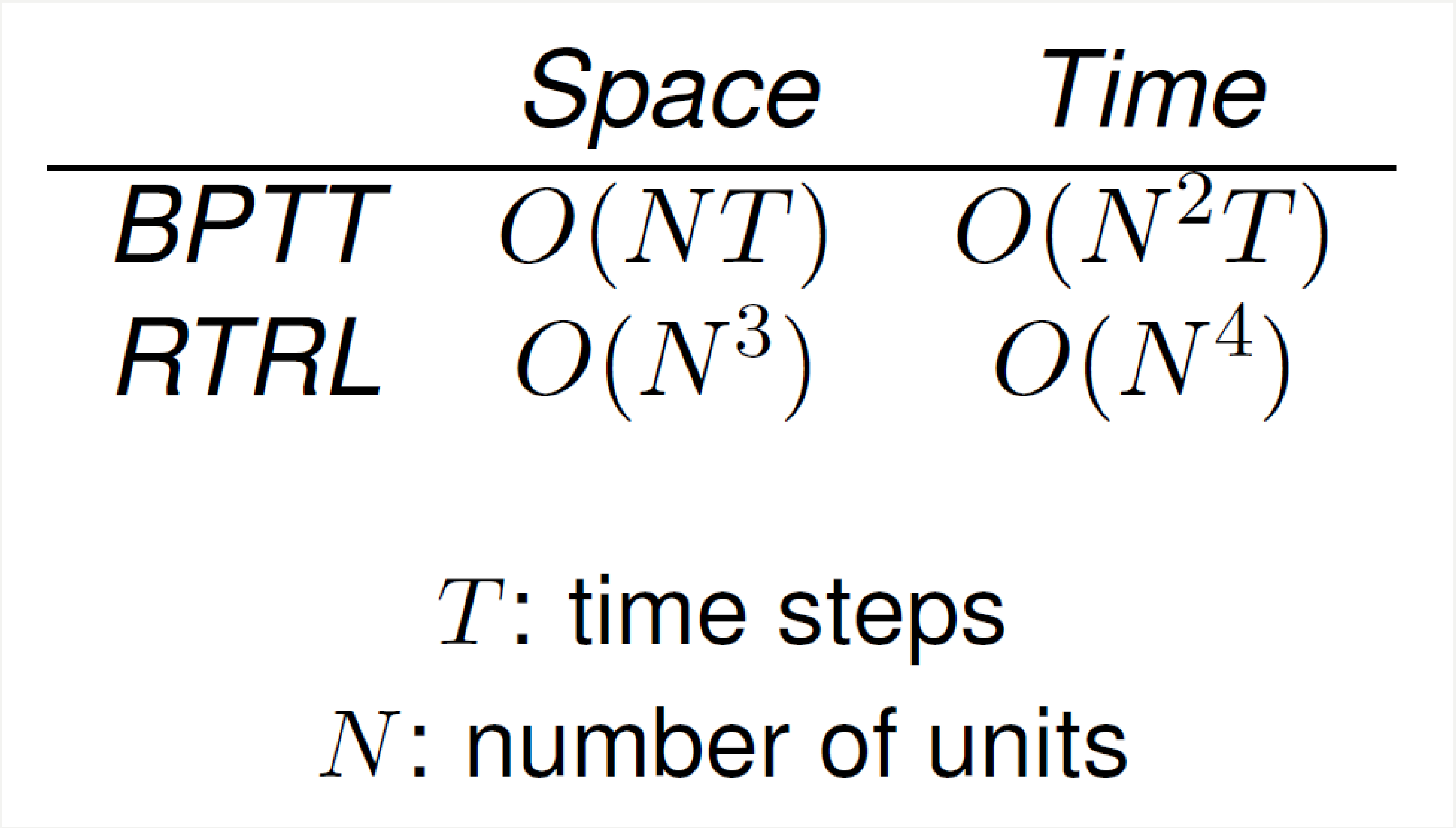


$\mathbf{o}_{t-2}$
$\mathbf{o}_{t-1}$
$\mathbf{o}_t$
$\frac{\partial \mathbf{o}_{t-2}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{o}_{t-1}}{\partial \mathbf{h}_{t-1}}$
$\frac{\partial \mathbf{o}_t}{\partial \mathbf{h}_t}$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{h}_{t-3}}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{h}_{t-2}}$
$\frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t-1}}$
$\mathbf{h}_{t-2}$
$\mathbf{h}_{t-1}$
$\mathbf{h}_t$
$\frac{\partial \mathbf{h}_{t-2}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_{t-1}}{\partial \mathbf{W}_x}$
$\frac{\partial \mathbf{h}_t}{\partial \mathbf{W}_x}$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{W}_x$
$\mathbf{x}_{t-2}$
$\mathbf{x}_{t-1}$
$\mathbf{x}_t$

Comparison of BPTT and RTRL

Both BPTT and RTRL compute the same gradients but in different ways.

They differ in computational complexity:



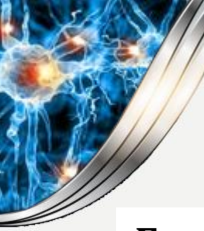
	Space	*Time*
BPTT	$O(NT)$	$O(N^2T)$
RTRL	$O(N^3)$	$O(N^4)$

T: time steps

N: number of units

Vectorization

What can we do to speed up computations and use parallel operations and GPU?



Gradient Descent for Training Dataset

For training dataset consisting of m training examples, we minimize the cost function J:

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} L\left(a^{(i)}, y^{(i)}\right)$$

$$\hat{y}^{(i)} = a^{(i)} = \sigma\left(z^{(i)}\right) = \sigma\left(w^T x^{(i)} + b\right)$$

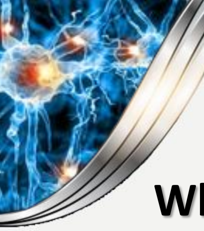
$$\frac{dJ(w,b)}{dw_j} = \frac{1}{m}\sum_{i=1}^{m} \frac{dL\left(a^{(i)}, y^{(i)}\right)}{dw_j} = \frac{1}{m}\sum_{i=1}^{m}\left(a^{(i)} - y^{(i)}\right)\cdot x_j^{(i)}$$

$$\frac{dJ(w,b)}{db} = \frac{1}{m}\sum_{i=1}^{m} \frac{dL\left(a^{(i)}, y^{(i)}\right)}{db} = \frac{1}{m}\sum_{i=1}^{m}\left(a^{(i)} - y^{(i)}\right)$$

```
repeat
  $J = 1$
  for $j = 1$ to $n_x$
    $dJw_j = 0$
  $dLb = 0$
  for $i = 1$ to $m$
    $z^{(i)} = w^T x^{(i)} + b$
    $a^{(i)} = \sigma\left(z^{(i)}\right)$
    $J += -\left(y^{(i)} \log a^{(i)} + \left(1 - y^{(i)}\right) \log\left(1 - a^{(i)}\right)\right)$
    $dJz^{(i)} = a^{(i)} - y^{(i)}$
    for $j = 1$ to $n_x$
      $dJw_j += x_j^{(i)} \cdot dJz^{(i)}$
    $dJb += dJz^{(i)}$
  $J /= m$
  for $j = 1$ to $n_x$
    $dJw_j /= m$
    $w_j -= \alpha \cdot dJw_j$
  $dJb /= m$
  $b -= \alpha \cdot dJb$
until $J < \varepsilon$
```

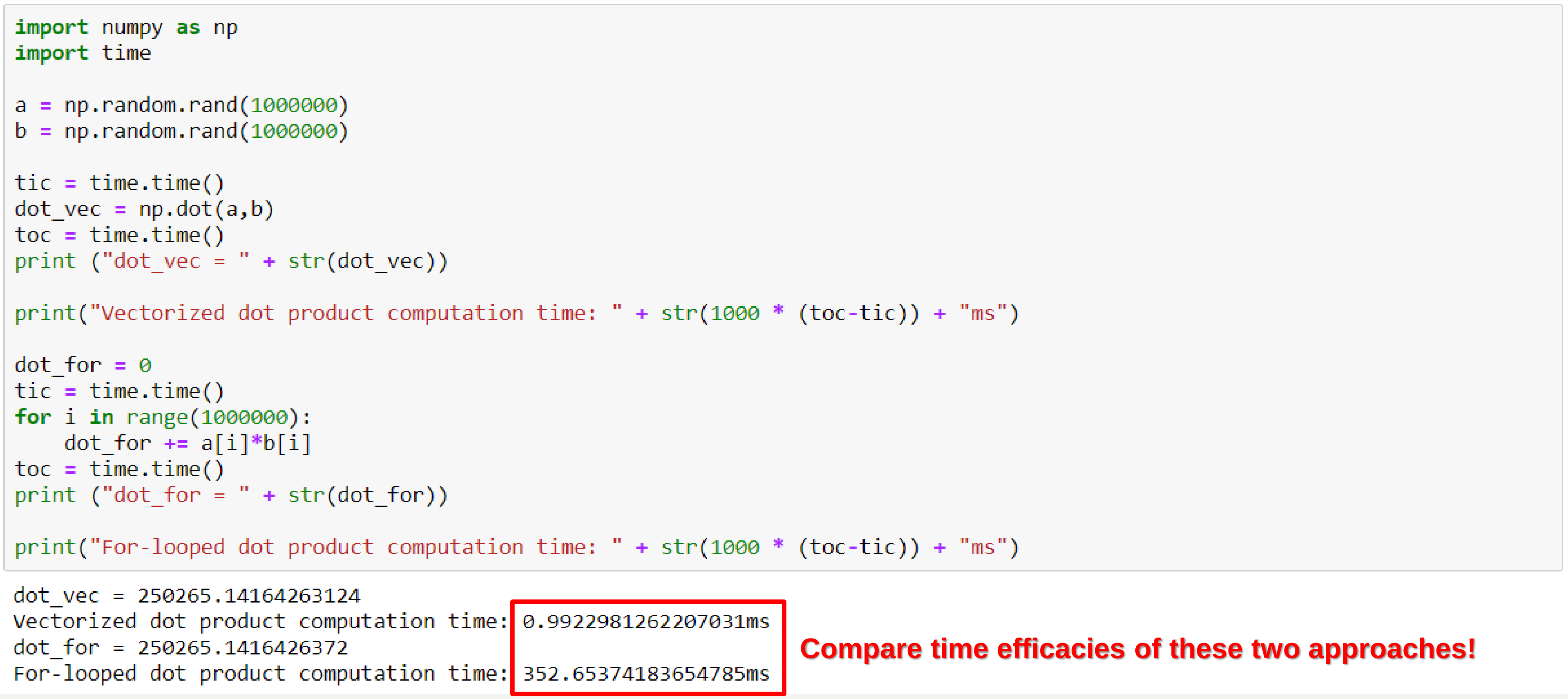
The final logistic regression gradient descent algorithm will repeatedly go through all training examples updating parameters until the cost function is not small enough.

To speed up computation we should use vectorization instead of for-loops.



Efficiency of Vectorization

When dealing with big data collections and big data vectors, we definitely should use vectorization (that performs SIMD operations) to proceed computations faster:



```
import numpy as np
import time

a = np.random.rand(1000000)
b = np.random.rand(1000000)

tic = time.time()
dot_vec = np.dot(a,b)
toc = time.time()
print ("dot_vec = " + str(dot_vec))

print("Vectorized dot product computation time: " + str(1000 * (toc-tic)) + "ms")

dot_for = 0
tic = time.time()
for i in range(1000000):
    dot_for += a[i]*b[i]
toc = time.time()
print ("dot_for = " + str(dot_for))

print("For-looped dot product computation time: " + str(1000 * (toc-tic)) + "ms")
```

```
dot_vec = 250265.14164263124
Vectorized dot product computation time: 0.9922981262207031ms
dot_for = 250265.1416426372
For-looped dot product computation time: 352.65374183654785ms
```

Compare time efficacies of these two approaches!

Conclusion:

Whenever possible, avoid explicit for-loops and use vectorization: np.dot(w.T,x), np.dot(W,x), np.multiply(x1,x2), np.outer(x1,x2), np.log(v), np.exp(v), np.abs(v), np.zeros(v), np.sum(v), np.max(v), np.min(v) etc. Vectorization uses parallel CPU or GPU operations (called SIMD – single instruction multiple data) proceed on parallelly working cores.

Vectorization of the Logistic Regression

Let's vectorize the previous algorithm:

```
repeat
  $J = 1$
  for $j = 1$ to $n_x$
    $dJw_j = 0$
  $dLb = 0$
  for $i = 1$ to $m$
    $z^{(i)} = w^T x^{(i)} + b$
    $a^{(i)} = \sigma\left(z^{(i)}\right)$
    $J += -\left(y^{(i)} \log a^{(i)} + \left(1 - y^{(i)}\right) \log\left(1 - a^{(i)}\right)\right)$
    $dJz^{(i)} = a^{(i)} - y^{(i)}$
    for $j = 1$ to $n_x$
      $dJw_j += x_j^{(i)} \cdot dJz^{(i)}$
    $dJb += dJz^{(i)}$
  $J /= m$
  for $j = 1$ to $n_x$
    $dJw_j /= m$
    $w_j -= \alpha \cdot dJw_j$
  $dJb /= m$
  $b -= \alpha \cdot dJb$
until $J < \varepsilon$
```

$$dJw = np.zeros\left(\left(n_x, 1\right)\right)$$

broadcasted

$$Z = w^T X + b = np.dot(w.T, X) + \boxed{b}$$

We use matrices

$$X = \left[x^{(1)}, x^{(2)}, \dots, x^{(m)}\right]$$

$$Z = \left[z^{(1)}, z^{(2)}, \dots, z^{(m)}\right]$$

$$A = \left[a^{(1)}, a^{(2)}, \dots, a^{(m)}\right]$$

$$Y = \left[y^{(1)}, y^{(2)}, \dots, y^{(m)}\right]$$

$$A = \sigma(Z)$$

$$dJZ = A - Y$$

$$dJw += x^{(i)} \cdot dJz^{(i)}$$

$$dJw = \frac{1}{m} X \cdot dJZ^T$$

$$dJb = \frac{1}{m} \cdot np.sum(dJZ)$$

$$dJw /= m$$

$$w -= \alpha \cdot dJw$$

$$b -= \alpha \cdot dJb$$

Broadcasting

How can we multiply data to use different shapes of structures which do not fit.

Broadcasting in Python

Broadcasting stands for a special operation which multiplies the data in rows and/or columns to fit the size of a bigger structure and allow to perform operations:

BROADCASTING PRINCIPLE:

(m, n)	+	(1, n) → (m, n)	=	(m, n)
(m, n)	-	(1, n) → (m, n)	=	(m, n)
(m, n)	*	(1, n) → (m, n)	=	(m, n)
(m, n)	/	(1, n) → (m, n)	=	(m, n)
(m, n)	+	(m, 1) → (m, n)	=	(m, n)
(m, n)	-	(m, 1) → (m, n)	=	(m, n)
(m, n)	*	(m, 1) → (m, n)	=	(m, n)
(m, n)	/	(m, 1) → (m, n)	=	(m, n)

BROADCASTING SAMPLES:

$$\begin{bmatrix}1\\2\\3\end{bmatrix} + 10 = \begin{bmatrix}11\\12\\13\end{bmatrix}$$

where 10 was broadcasted (1,1) → (4,1)

$$\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}10\\10\\10\end{bmatrix} = \begin{bmatrix}11\\12\\13\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}10 & 20 & 30\end{bmatrix} = \begin{bmatrix}11 & 22 & 33\\14 & 25 & 36\end{bmatrix}$$

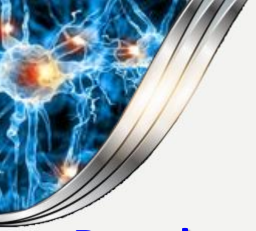
where $\begin{bmatrix}10 & 20 & 30\end{bmatrix}$ was broadcasted (1,3) → (2,3)

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}10 & 20 & 30\\10 & 20 & 30\end{bmatrix} = \begin{bmatrix}11 & 22 & 33\\14 & 25 & 36\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}10\\20\end{bmatrix} = \begin{bmatrix}11 & 12 & 13\\24 & 25 & 26\end{bmatrix}$$

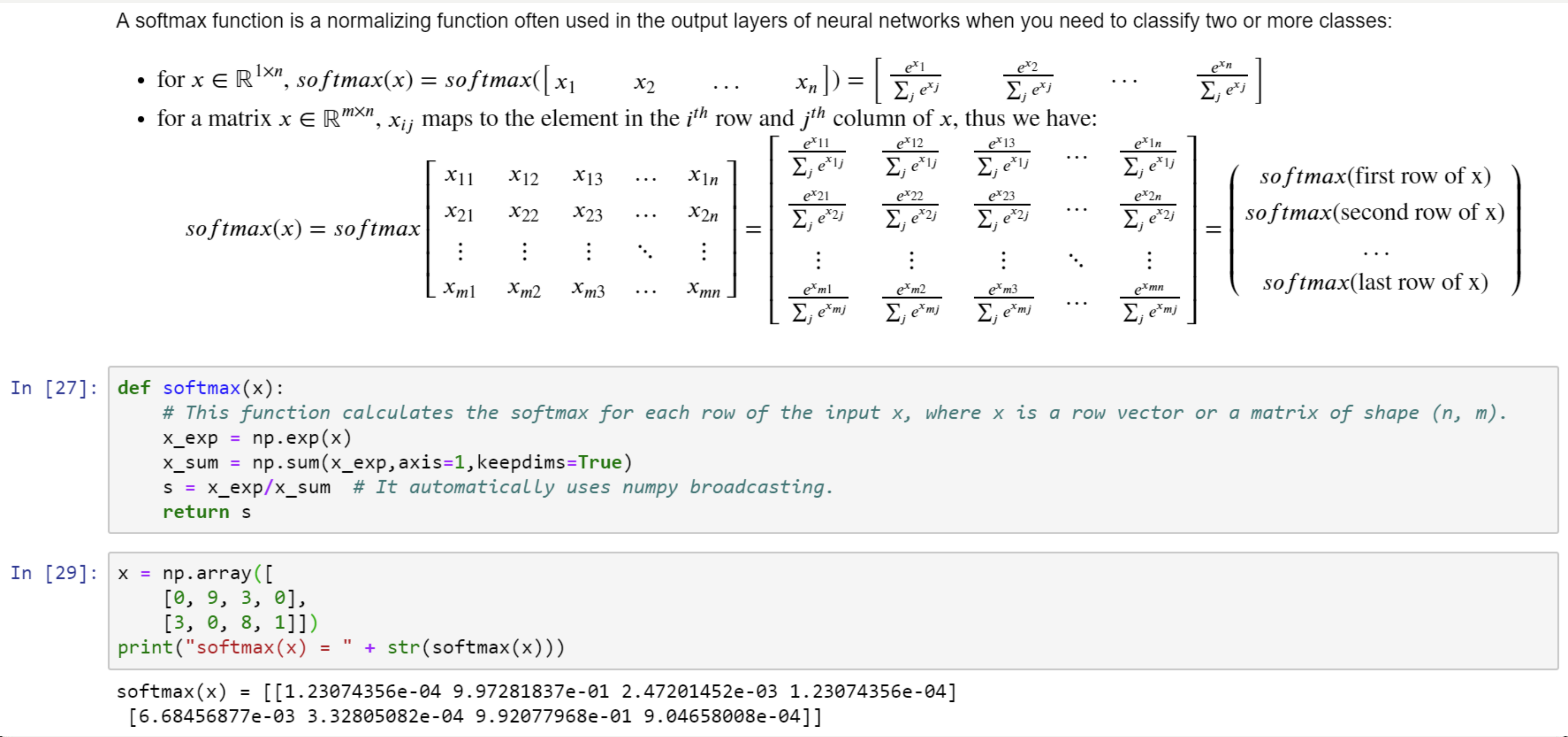
where $\begin{bmatrix}10\\20\end{bmatrix}$ was broadcasted (2,1) → (2,3)

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}10 & 10 & 10\\20 & 20 & 20\end{bmatrix} = \begin{bmatrix}11 & 12 & 13\\24 & 25 & 26\end{bmatrix}$$



Broadcasting in numpy

Broadcasting is very useful for performing mathematical operations between arrays of different shapes. The example below show the normalization of the data.



A softmax function is a normalizing function often used in the output layers of neural networks when you need to classify two or more classes:

- for $x \in \mathbb{R}^{1\times n}$, $softmax(x) = softmax(\begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}) = \begin{bmatrix} \frac{e^{x_1}}{\sum_j e^{x_j}} & \frac{e^{x_2}}{\sum_j e^{x_j}} & \dots & \frac{e^{x_n}}{\sum_j e^{x_j}} \end{bmatrix}$
- for a matrix $x \in \mathbb{R}^{m\times n}$, x_{ij} maps to the element in the i^{th} row and j^{th} column of x, thus we have:

$$softmax(x) = softmax\begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix} = \begin{bmatrix} \frac{e^{x_{11}}}{\sum_j e^{x_{1j}}} & \frac{e^{x_{12}}}{\sum_j e^{x_{1j}}} & \frac{e^{x_{13}}}{\sum_j e^{x_{1j}}} & \dots & \frac{e^{x_{1n}}}{\sum_j e^{x_{1j}}} \\ \frac{e^{x_{21}}}{\sum_j e^{x_{2j}}} & \frac{e^{x_{22}}}{\sum_j e^{x_{2j}}} & \frac{e^{x_{23}}}{\sum_j e^{x_{2j}}} & \dots & \frac{e^{x_{2n}}}{\sum_j e^{x_{2j}}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{e^{x_{m1}}}{\sum_j e^{x_{mj}}} & \frac{e^{x_{m2}}}{\sum_j e^{x_{mj}}} & \frac{e^{x_{m3}}}{\sum_j e^{x_{mj}}} & \dots & \frac{e^{x_{mn}}}{\sum_j e^{x_{mj}}} \end{bmatrix} = \begin{pmatrix} softmax\text{(first row of x)} \\ softmax\text{(second row of x)} \\ \dots \\ softmax\text{(last row of x)} \end{pmatrix}$$

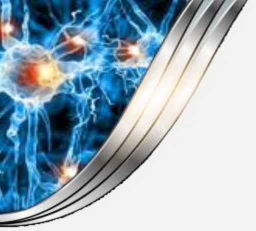
In [27]:
```python
def softmax(x):
    # This function calculates the softmax for each row of the input x, where x is a row vector or a matrix of shape (n, m).
    x_exp = np.exp(x)
    x_sum = np.sum(x_exp,axis=1,keepdims=True)
    s = x_exp/x_sum  # It automatically uses numpy broadcasting.
    return s
```

In [29]:
```python
x = np.array([
    [0, 9, 3, 0],
    [3, 0, 8, 1]])
print("softmax(x) = " + str(softmax(x)))
```

```
softmax(x) = [[1.23074356e-04 9.97281837e-01 2.47201452e-03 1.23074356e-04]
 [6.68456877e-03 3.32805082e-04 9.92077968e-01 9.04658008e-04]]
```

Shapes of Matrices

What shapes of matrices do we use and how can we reshape them?

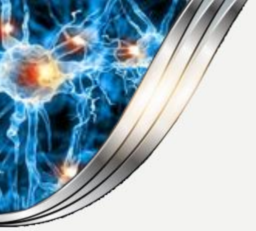


Lists vs. Vectors and Matrices

```python
import numpy as np

print("List of values:")
a = np.random.randn(6)   # generates list of samples from the normal distribution, while rand from unifrom (in range [0,1))
print(a)
print(a.shape)           # the shape suggest that a is a list
print(a.T)               # the list cannot be transposed because it is not a vector or matrix!
print(np.dot(a,a.T))     # what should it mean?!

print("Vector of values:")
b = np.random.randn(6,1) # generates matrix of samples from the normal distribution
print(b)
print(b.shape)           # the shape suggest that b is a matrix (vector)
print(b.T)               # the vector can be transposed
print(np.dot(b,b.T))     # now we get a matrix as a result of multiplication of the vectors
```

Be careful when creating vectors because lists have no shape and are declared similarly.

```
List of values:
[ 1.63130571  1.30039595 -1.42170758  1.28012586  1.63085575  0.64436582]
(6,)
[ 1.63130571  1.30039595 -1.42170758  1.28012586  1.63085575  0.64436582]
11.08706038339276
Vector of values:
[[-1.2426375 ]
 [-0.54254535]
 [ 0.76000053]
 [-0.83861851]
 [ 0.66463   ]
 [-1.60972555]]
(6, 1)
[[-1.2426375  -0.54254535  0.76000053 -0.83861851  0.66463    -1.60972555]]
[[ 1.54414796  0.6741872  -0.94440516  1.04209881 -0.82589416  2.00030533]
 [ 0.6741872   0.29435546 -0.41233475  0.45498857 -0.36059191  0.87334911]
 [-0.94440516 -0.41233475  0.57760081 -0.63735051  0.50511915 -1.22339227]
 [ 1.04209881  0.45498857 -0.63735051  0.703281   -0.55737102  1.34994564]
 [-0.82589416 -0.36059191  0.50511915 -0.55737102  0.44173303 -1.06987188]
 [ 2.00030533  0.87334911 -1.22339227  1.34994564 -1.06987188  2.59121633]]
```



Column and Row Vectors

```python
import numpy as np

C=np.random.randn(5,1)
D=np.random.randn(1,5)
print("We define matrices and vectors using (m, n) where m is a number of rows, and n is a number of columns")
print(C)
print("... is a column vector")
print(D)
print("... is a row vector")
```

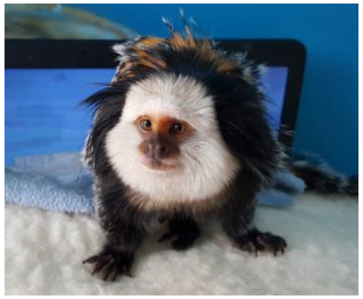
```
We define matrices and vectors using (m, n) where m is a number of rows, and n is a number of columns
[[ 0.23665149]
 [ 0.45132428]
 [-0.89728231]
 [ 0.72912635]
 [-0.92627707]]
... is a column vector
[[ 0.99318971 -0.8439588   1.20413677 -1.00233032 -1.55317979]]
... is a row vector
```

```python
import numpy as np

a = np.random.randn(5)    # the list can be reshaped to create a vector
print(a)
print(a.shape)
a = a.reshape((5,1))
print(a)
print(a.shape)

assert(a.shape == (5, 1)) # we can check whether the shape is correct
```

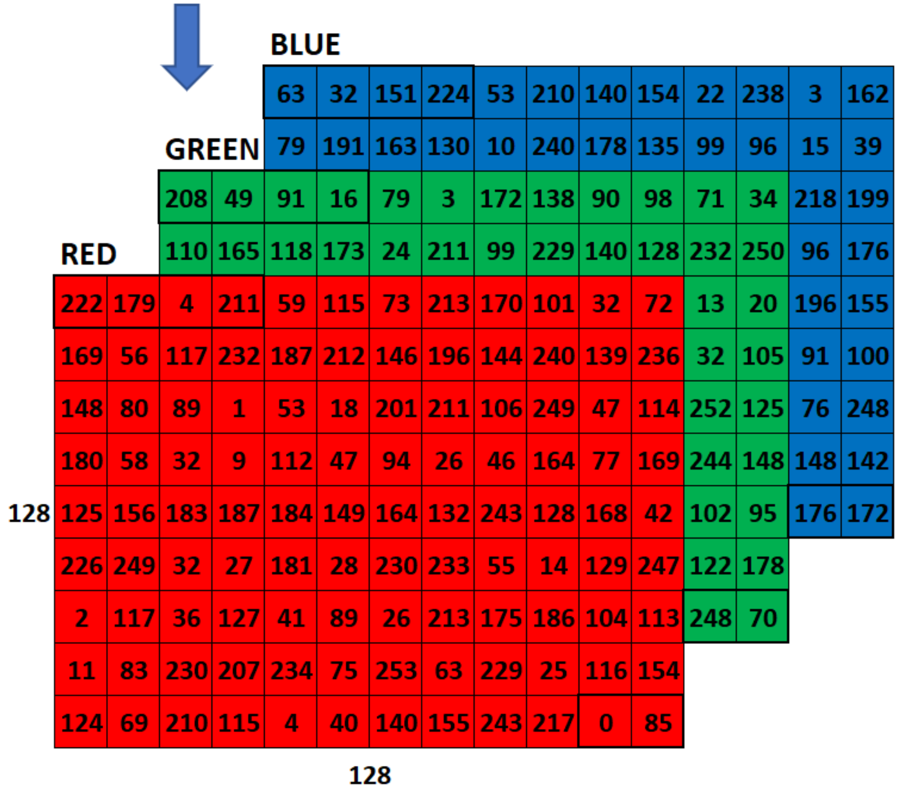
```
[-0.07161977 -2.17009596  0.09644837  0.5044574  -0.04263376]
(5,)
[[-0.07161977]
 [-2.17009596]
 [ 0.09644837]
 [ 0.5044574 ]
 [-0.04263376]]
(5, 1)
```

Reshaping Image Matrices

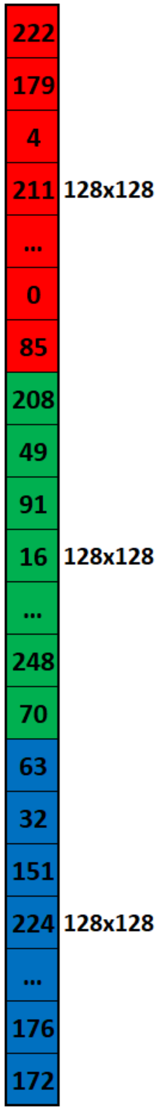
When working with images in deep learning,



Images are reprezented as a combination of three colours reprezented by three matrices that store the intensities of these colours (Red, Green, and Blue):



$\mathbf{x}$ $n_x = 128\text{x}128\text{x}3 = 49152$ is the dimension of vector x



In the binary classification tasks, input vectors are assigned to one of the two classes 0 or 1 that is the output value y of the classification process.

So, we have to create the transformation

$x \rightarrow y$

and denote the training example as pairs (x, y)

where

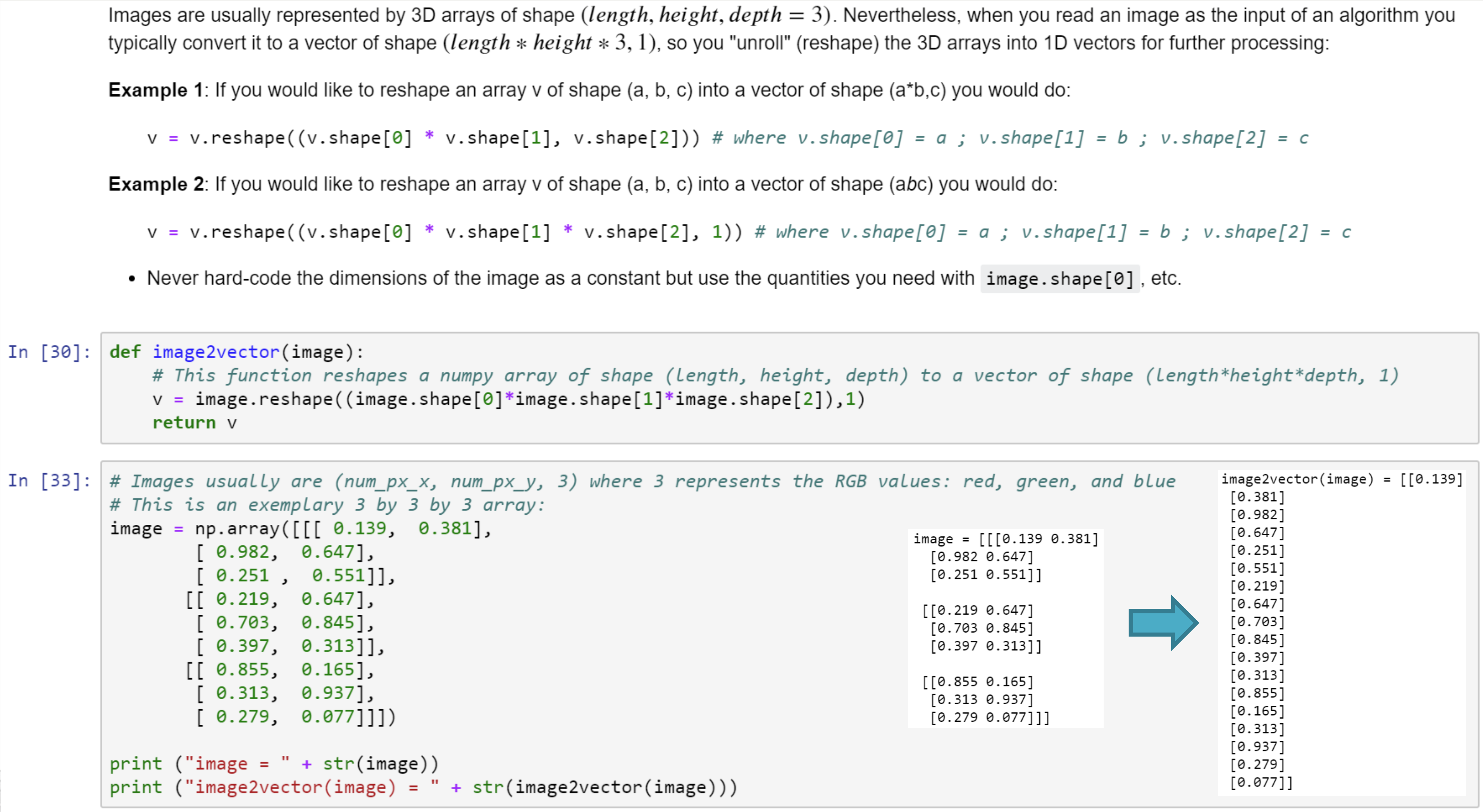
$x \in R^{n_x}$

$y \in \{0, 1\}$

we typically reshape them into vector representation using np.reshape().

Shape and Reshape Vectors and Matrices

We use the numpy functions np.shape() and np.reshape() in deep learning:



Images are usually represented by 3D arrays of shape $(length, height, depth = 3)$. Nevertheless, when you read an image as the input of an algorithm you typically convert it to a vector of shape $(length * height * 3, 1)$, so you "unroll" (reshape) the 3D arrays into 1D vectors for further processing:

Example 1: If you would like to reshape an array v of shape (a, b, c) into a vector of shape (a*b,c) you would do:

```
v = v.reshape((v.shape[0] * v.shape[1], v.shape[2])) # where v.shape[0] = a ; v.shape[1] = b ; v.shape[2] = c
```

Example 2: If you would like to reshape an array v of shape (a, b, c) into a vector of shape (abc) you would do:

```
v = v.reshape((v.shape[0] * v.shape[1] * v.shape[2], 1)) # where v.shape[0] = a ; v.shape[1] = b ; v.shape[2] = c
```

- Never hard-code the dimensions of the image as a constant but use the quantities you need with `image.shape[0]`, etc.

In [30]:
```
def image2vector(image):
    # This function reshapes a numpy array of shape (length, height, depth) to a vector of shape (length*height*depth, 1)
    v = image.reshape((image.shape[0]*image.shape[1]*image.shape[2]),1)
    return v
```

In [33]:
```
# Images usually are (num_px_x, num_px_y, 3) where 3 represents the RGB values: red, green, and blue
# This is an exemplary 3 by 3 by 3 array:
image = np.array([[[ 0.139,  0.381],
        [ 0.982,  0.647],
        [ 0.251 ,  0.551]],
       [[ 0.219,  0.647],
        [ 0.703,  0.845],
        [ 0.397,  0.313]],
       [[ 0.855,  0.165],
        [ 0.313,  0.937],
        [ 0.279,  0.077]]])

print ("image = " + str(image))
print ("image2vector(image) = " + str(image2vector(image)))
```

```
image = [[[0.139 0.381]
  [0.982 0.647]
  [0.251 0.551]]

 [[0.219 0.647]
  [0.703 0.845]
  [0.397 0.313]]

 [[0.855 0.165]
  [0.313 0.937]
  [0.279 0.077]]]
```

```
image2vector(image) = [[0.139]
 [0.381]
 [0.982]
 [0.647]
 [0.251]
 [0.551]
 [0.219]
 [0.647]
 [0.703]
 [0.845]
 [0.397]
 [0.313]
 [0.855]
 [0.165]
 [0.313]
 [0.937]
 [0.279]
 [0.077]]
```

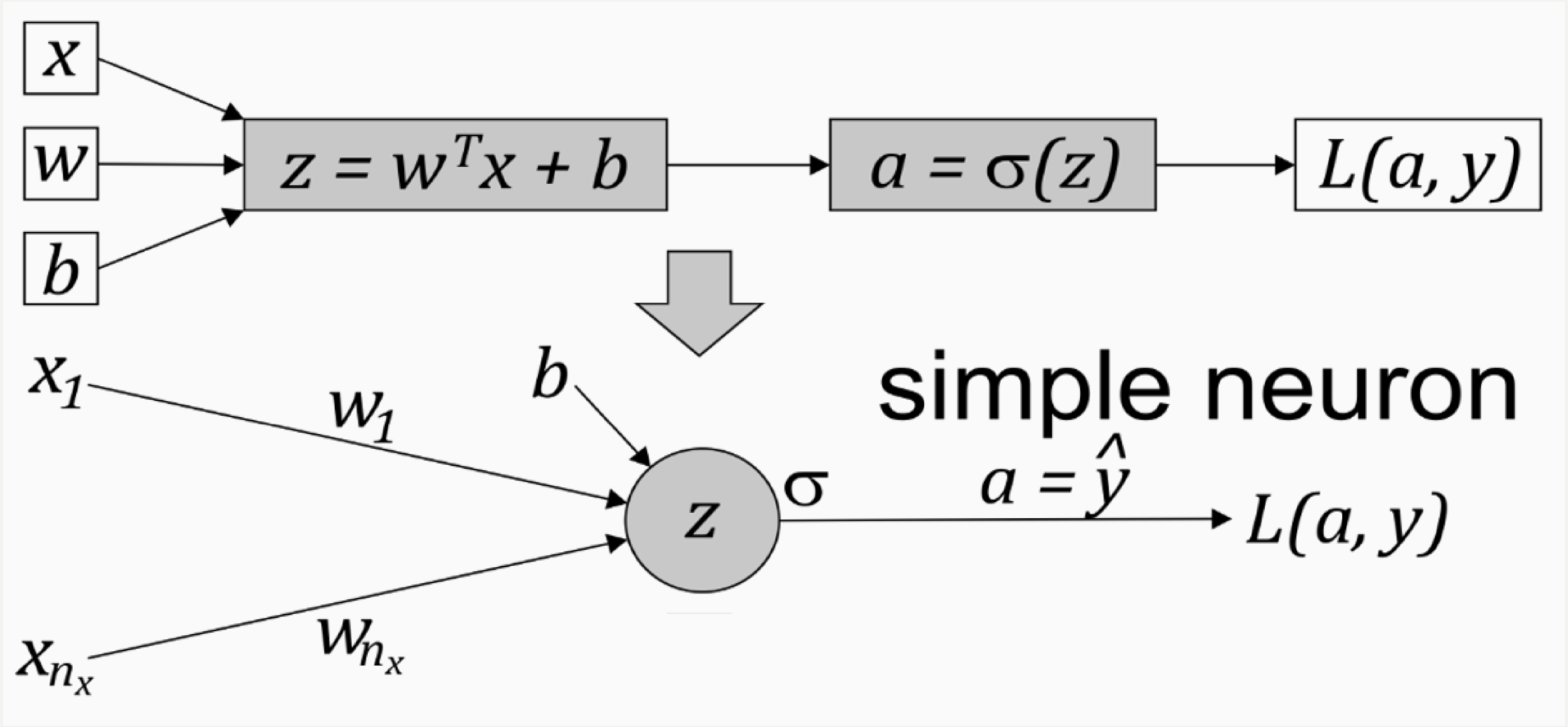
- **X.shape is used to get the shape (dimension) of a vector or a matrix X.**
- **X.reshape(...) is used to reshape a vector or a matrix X into some other dimension(s).**

Simple Network Construction

Construction of the network using stacked vectors and matrices – how do we do it?

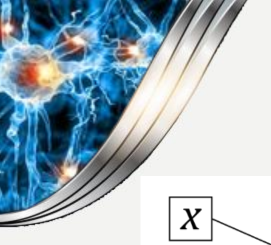
Simple Neuron Definition

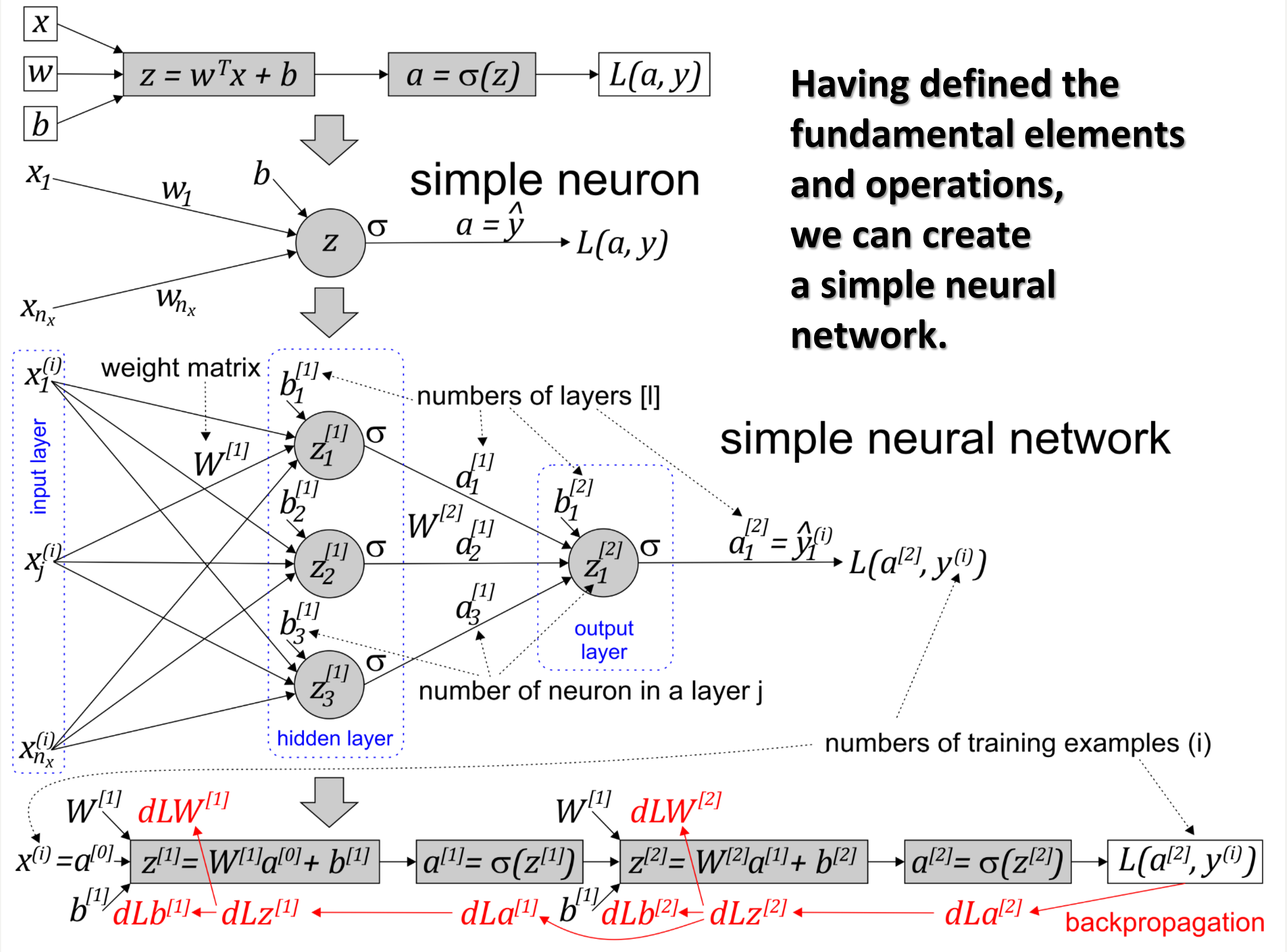
We defined the fundamental elements and operations on a single neuron as a weighted sum of inputs plus bias and the result is used to calculate the output using an activation function (here a sigmoid function).



The achieved output is used to calculate the loss and corrections.

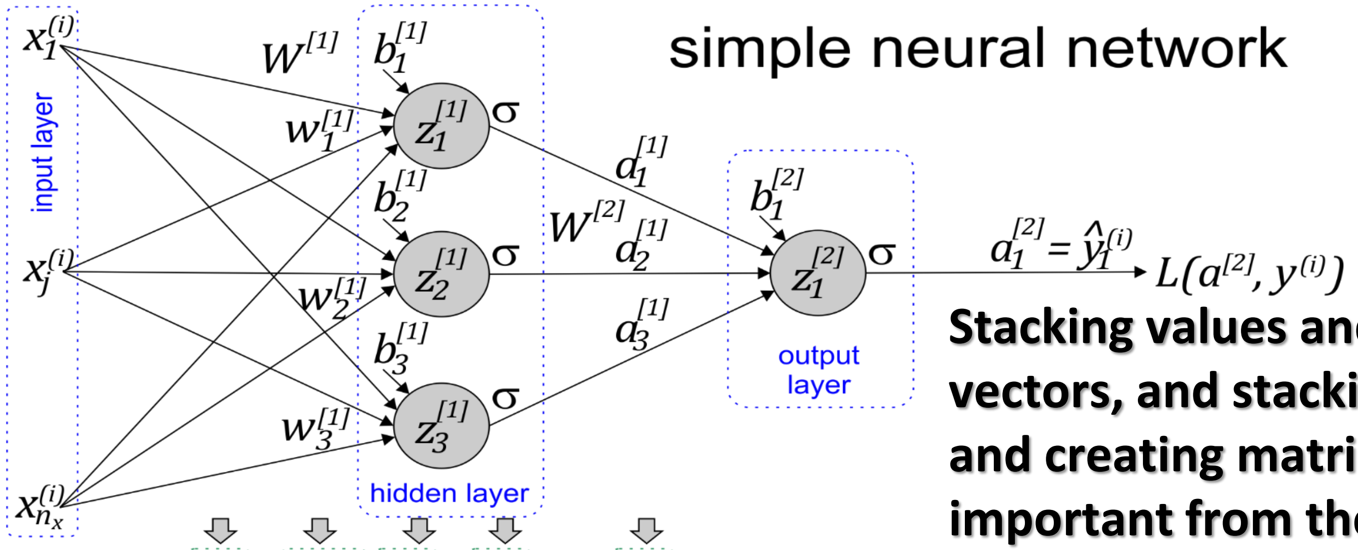
Simple Neural Network





Having defined the fundamental elements and operations, we can create a simple neural network.

Stacking Neurons Vertically and Vectorizing



$$z_1^{[1]} = w_1^{[1]T} a^{[0]} + b_1^{[1]} \longrightarrow a_1^{[1]} = \sigma(z_1^{[1]})$$
$$z_2^{[1]} = w_2^{[1]T} a^{[0]} + b_2^{[1]} \longrightarrow a_2^{[1]} = \sigma(z_2^{[1]})$$
$$z_3^{[1]} = w_3^{[1]T} a^{[0]} + b_3^{[1]} \longrightarrow a_3^{[1]} = \sigma(z_3^{[1]})$$

$$\Downarrow$$

$$\begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \end{bmatrix} \begin{bmatrix} w_1^{[1]T} \\ w_2^{[1]T} \\ w_3^{[1]T} \end{bmatrix} \begin{bmatrix} a_1^{[0]} \\ a_2^{[0]} \\ a_3^{[0]} \end{bmatrix} \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix} \qquad \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \end{bmatrix}$$

$$\Downarrow$$

$$z^{[1]} \quad W^{[1]} \quad a^{[0]} \quad b^{[1]} \qquad a^{[1]}$$

$$\Downarrow$$

$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]} \longrightarrow a^{[1]} = \sigma(z^{[1]})$$

Stacking values and creating vectors, and stacking vectors and creating matrices is very important from the efficiency of computation point of view because it allows to use parallel operations of GPU!

numbers of layers [l]

numbers of training examples (i)

number of neuron in a layer j

Stacking Examples Horizontally and Vectorizing

$$
\begin{array}{cc}
 & \text{No of neurons of (l-1)-th layer} \\
W^{[l]} = & \begin{bmatrix} — & w_1^{[l]T} & — \\ — & w_2^{[l]T} & — \\ & \vdots & \\ — & w_{n^{[l]}}^{[l]T} & — \end{bmatrix} \text{No of neurons of l-th layer}
\end{array}
$$

$$z_1^{[1](i)} = w_1^{[1]T} a^{[0](i)} + b_1^{[1]} \longrightarrow a_1^{[1](i)} = \sigma(z_1^{[1](i)})$$

$$z_2^{[1](i)} = w_2^{[1]T} a^{[0](i)} + b_2^{[1]} \longrightarrow a_2^{[1](i)} = \sigma(z_2^{[1](i)})$$

$$z_3^{[1](i)} = w_3^{[1]T} a^{[0](i)} + b_3^{[1]} \longrightarrow a_3^{[1](i)} = \sigma(z_3^{[1](i)})$$

<span>Stacking</span> vectors of training examples horizontally creating matrices is very important from the efficiency of computation point of view!

$$\begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \\ x_3^{(i)} \end{bmatrix} = \begin{bmatrix} a_1^{[0](i)} \\ a_2^{[0](i)} \\ a_3^{[0](i)} \end{bmatrix} \text{input examples}
\qquad
\begin{bmatrix} z_1^{[1](i)} \\ z_2^{[1](i)} \\ z_3^{[1](i)} \end{bmatrix}
\begin{bmatrix} w_1^{[1]T} \\ w_2^{[1]T} \\ w_3^{[1]T} \end{bmatrix}
\begin{bmatrix} a_1^{[0](i)} \\ a_2^{[0](i)} \\ a_3^{[0](i)} \end{bmatrix}
\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix}
\begin{bmatrix} a_1^{[1](i)} \\ a_2^{[1](i)} \\ a_3^{[1](i)} \end{bmatrix}$$

numbers of layers [l]

numbers of training examples (i)

number of neuron in a layer j

$$x^{(i)} = a^{[0](i)} \qquad z^{[1](i)} \quad W^{[1]} \quad a^{[0](i)} \quad b^{[1]} \quad a^{[1](i)}$$

$$\begin{matrix} x^{(1)} \\ \vdots \\ x^{(i)} \\ \vdots \\ x^{(m)} \end{matrix}
\qquad
\begin{array}{cc}
\text{training examples} & \\
\begin{bmatrix} | & & | & & | \\ x^{(1)} & \dots & x^{(i)} & \dots & x^{(m)} \\ | & & | & & | \end{bmatrix} & \text{input data dimension}
\end{array}$$

$$z^{[1](1)} = W^{[1]} a^{[0](1)} + b^{[1]} \longrightarrow a^{[1](1)} = \sigma(z^{[1](1)})$$

$$\vdots$$

$$z^{[1](i)} = W^{[1]} a^{[0](i)} + b^{[1]} \longrightarrow a^{[1](i)} = \sigma(z^{[1](i)})$$

$$\vdots$$

$$z^{[1](m)} = W^{[1]} a^{[0](m)} + b^{[1]} \longrightarrow a^{[1](m)} = \sigma(z^{[1](m)})$$

$$\boxed{X = A^{[0]} \quad \textbf{After Vectorizing} \quad Z^{[1]} = W^{[1]} A^{[0]} + b^{[1]} \longrightarrow A^{[1]} = \sigma(Z^{[1]})}$$

$$\begin{array}{cc}
\text{training examples} & \\
\begin{bmatrix} | & & | & & | \\ a^{[l](1)} & \dots & a^{[l](i)} & \dots & a^{[l](m)} \\ | & & | & & | \end{bmatrix} & \text{hidden neurons}
\end{array}$$

$$\begin{array}{cc}
\text{training examples} & \\
\begin{bmatrix} | & & | & & | \\ z^{[l](1)} & \dots & z^{[l](i)} & \dots & z^{[l](m)} \\ | & & | & & | \end{bmatrix} & \text{hidden neurons}
\end{array}$$

Vectorized Operations

How does vectorization speed up computations?

Vectorization of Dot Product

In deep learning, you deal with very large datasets. Non-computationally-optimal functions become a huge bottleneck in your algorithms and can result in models that take ages to run. To make sure that your code is computationally efficient, you should use vectorization. Compare the following codes:

```
import time

x1 = [5, 1, 0, 3, 8, 2, 5, 6, 0, 1, 2, 5, 9, 0, 7]  # x1 = np.random.rand(1000000)
x2 = [2, 5, 2, 0, 3, 2, 2, 9, 1, 0, 2, 5, 4, 0, 9]  # x2 = np.random.rand(1000000)

### CLASSIC DOT PRODUCT OF VECTORS IMPLEMENTATION ###
tic = time.process_time()
dot = 0
for i in range(len(x1)):
    dot+= x1[i] * x2[i]
toc = time.process_time()
print ("for-looped dot = " + str(dot) + "\n ----- Computation time = " + str(1000*(toc - tic)) + "ms")

### VECTORIZED DOT PRODUCT OF VECTORS ###
tic = time.process_time()
dot = np.dot(x1,x2)
toc = time.process_time()
print ("vectorized dot = " + str(dot) + "\n ----- Computation time = " + str(1000*(toc - tic)) + "ms")
```

```
for-looped dot = 235
 ----- Computation time = 0.0ms
vectorized dot = 235
 ----- Computation time = 0.0ms
```

Use more data to see the difference!

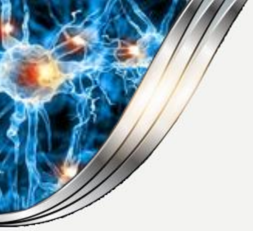
Vectorization of Outer Product

In deep learning, you deal with very large datasets. Non-computationally-optimal functions become a huge bottleneck in your algorithms and can result in models that take ages to run. To make sure that your code is computationally efficient, you should use vectorization. Compare the following codes:

```python
import time

x1 = [5, 1, 0, 3, 8, 2, 5, 6, 0, 1, 2, 5, 9, 0, 7]  # x1 = np.random.rand(1000000)
x2 = [2, 5, 2, 0, 3, 2, 2, 9, 1, 0, 2, 5, 4, 0, 9]  # x2 = np.random.rand(1000000)

### CLASSIC OUTER PRODUCT IMPLEMENTATION ###
tic = time.process_time()
outer = np.zeros((len(x1),len(x2))) # we create a len(x1)*len(x2) matrix with only zeros
for i in range(len(x1)):
    for j in range(len(x2)):
        outer[i,j] = x1[i] * x2[j]
toc = time.process_time()
print ("for-looped outer = " + str(outer) + "\n ----- Computation time = " + str(1000*(toc - tic)) + "ms")

### VECTORIZED OUTER PRODUCT ###
tic = time.process_time()
outer = np.outer(x1,x2)
toc = time.process_time()
print ("vectorized outer = " + str(outer) + "\n ----- Computation time = " + str(1000*(toc - tic)) + "ms")
```

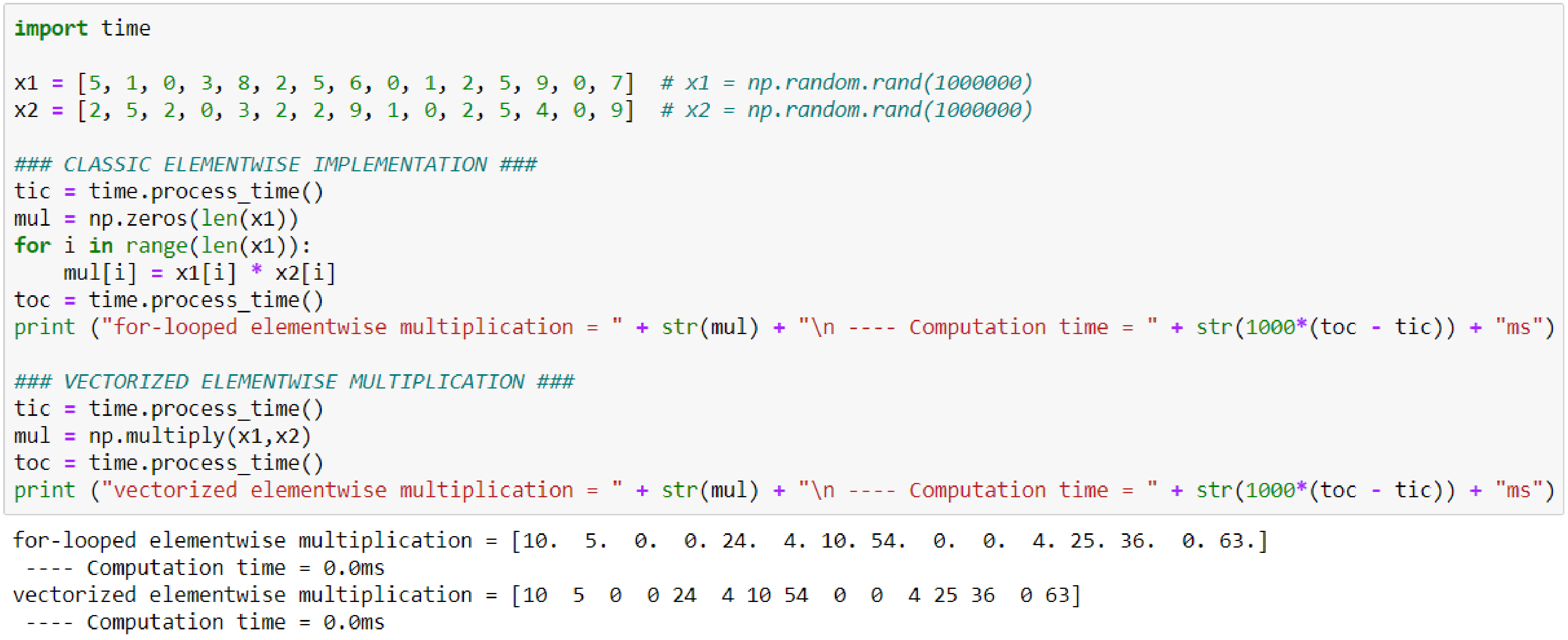
```
outer = [[81. 18. 18. 81.  0. 81. 18. 45.  0.  0. 81. 18. 45.  0.  0.]
 [18.  4.  4. 18.  0. 18.  4. 10.  0.  0. 18.  4. 10.  0.  0.]
 [45. 10. 10. 45.  0. 45. 10. 25.  0.  0. 45. 10. 25.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [63. 14. 14. 63.  0. 63. 14. 35.  0.  0. 63. 14. 35.  0.  0.]
 [45. 10. 10. 45.  0. 45. 10. 25.  0.  0. 45. 10. 25.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [81. 18. 18. 81.  0. 81. 18. 45.  0.  0. 81. 18. 45.  0.  0.]
 [18.  4.  4. 18.  0. 18.  4. 10.  0.  0. 18.  4. 10.  0.  0.]
 [45. 10. 10. 45.  0. 45. 10. 25.  0.  0. 45. 10. 25.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]]
 ----- Computation time = 0.0ms
```

```
outer = [[81 18 18 81  0 81 18 45  0  0 81 18 45  0  0]
 [18  4  4 18  0 18  4 10  0  0 18  4 10  0  0]
 [45 10 10 45  0 45 10 25  0  0 45 10 25  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]
 [63 14 14 63  0 63 14 35  0  0 63 14 35  0  0]
 [45 10 10 45  0 45 10 25  0  0 45 10 25  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]
 [81 18 18 81  0 81 18 45  0  0 81 18 45  0  0]
 [18  4  4 18  0 18  4 10  0  0 18  4 10  0  0]
 [45 10 10 45  0 45 10 25  0  0 45 10 25  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]
 [ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0]]
 ----- Computation time = 0.0ms
```



Vectorization of Element-Wise Multiplication

In deep learning, you deal with very large datasets. Non-computationally-optimal functions become a huge bottleneck in your algorithms and can result in models that take ages to run. To make sure that your code is computationally efficient, you should use vectorization. Compare the following codes:



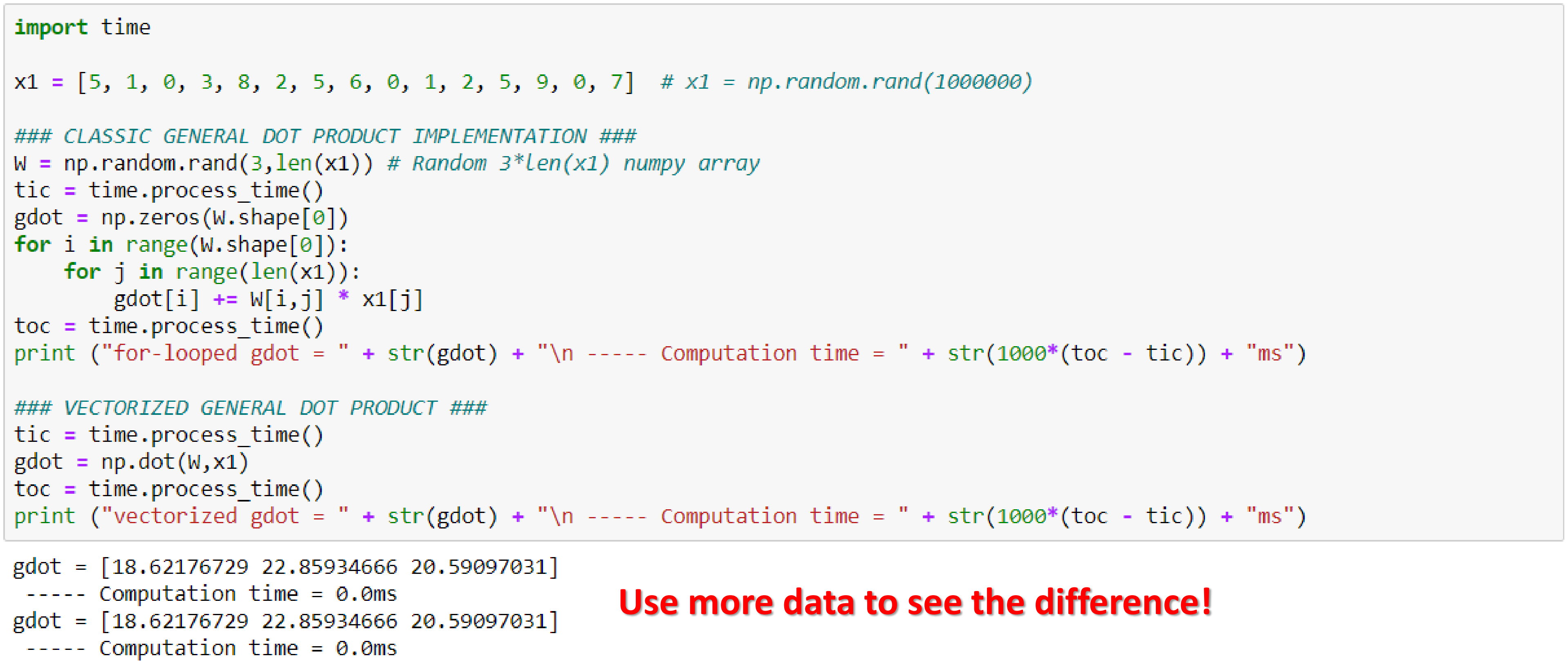
```python
import time

x1 = [5, 1, 0, 3, 8, 2, 5, 6, 0, 1, 2, 5, 9, 0, 7]  # x1 = np.random.rand(1000000)
x2 = [2, 5, 2, 0, 3, 2, 2, 9, 1, 0, 2, 5, 4, 0, 9]  # x2 = np.random.rand(1000000)

### CLASSIC ELEMENTWISE IMPLEMENTATION ###
tic = time.process_time()
mul = np.zeros(len(x1))
for i in range(len(x1)):
    mul[i] = x1[i] * x2[i]
toc = time.process_time()
print ("for-looped elementwise multiplication = " + str(mul) + "\n ---- Computation time = " + str(1000*(toc - tic)) + "ms")

### VECTORIZED ELEMENTWISE MULTIPLICATION ###
tic = time.process_time()
mul = np.multiply(x1,x2)
toc = time.process_time()
print ("vectorized elementwise multiplication = " + str(mul) + "\n ---- Computation time = " + str(1000*(toc - tic)) + "ms")
```

```
for-looped elementwise multiplication = [10.  5.  0.  0. 24.  4. 10. 54.  0.  0.  4. 25. 36.  0. 63.]
 ---- Computation time = 0.0ms
vectorized elementwise multiplication = [10  5  0  0 24  4 10 54  0  0  4 25 36  0 63]
 ---- Computation time = 0.0ms
```

Use more data to see the difference!

Vectorization of General Dot Product

In deep learning, you deal with very large datasets. Non-computationally-optimal functions become a huge bottleneck in your algorithms and can result in models that take ages to run. To make sure that your code is computationally efficient, you should use vectorization. Compare the following codes:



```python
import time

x1 = [5, 1, 0, 3, 8, 2, 5, 6, 0, 1, 2, 5, 9, 0, 7]  # x1 = np.random.rand(1000000)

### CLASSIC GENERAL DOT PRODUCT IMPLEMENTATION ###
W = np.random.rand(3,len(x1)) # Random 3*len(x1) numpy array
tic = time.process_time()
gdot = np.zeros(W.shape[0])
for i in range(W.shape[0]):
    for j in range(len(x1)):
        gdot[i] += W[i,j] * x1[j]
toc = time.process_time()
print ("for-looped gdot = " + str(gdot) + "\n ----- Computation time = " + str(1000*(toc - tic)) + "ms")

### VECTORIZED GENERAL DOT PRODUCT ###
tic = time.process_time()
gdot = np.dot(W,x1)
toc = time.process_time()
print ("vectorized gdot = " + str(gdot) + "\n ----- Computation time = " + str(1000*(toc - tic)) + "ms")
```

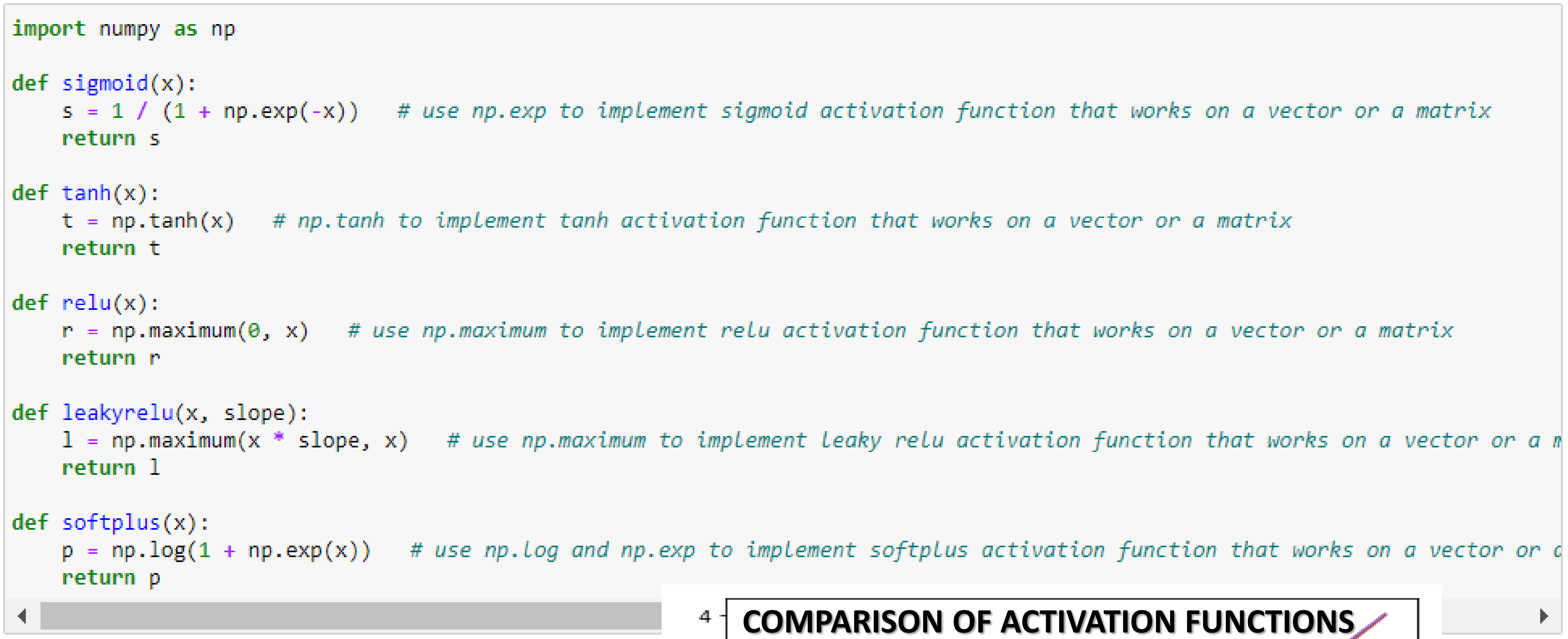
```
gdot = [18.62176729 22.85934666 20.59097031]
 ----- Computation time = 0.0ms
gdot = [18.62176729 22.85934666 20.59097031]
 ----- Computation time = 0.0ms
```

Use more data to see the difference!

Stacking and Training in Parallel

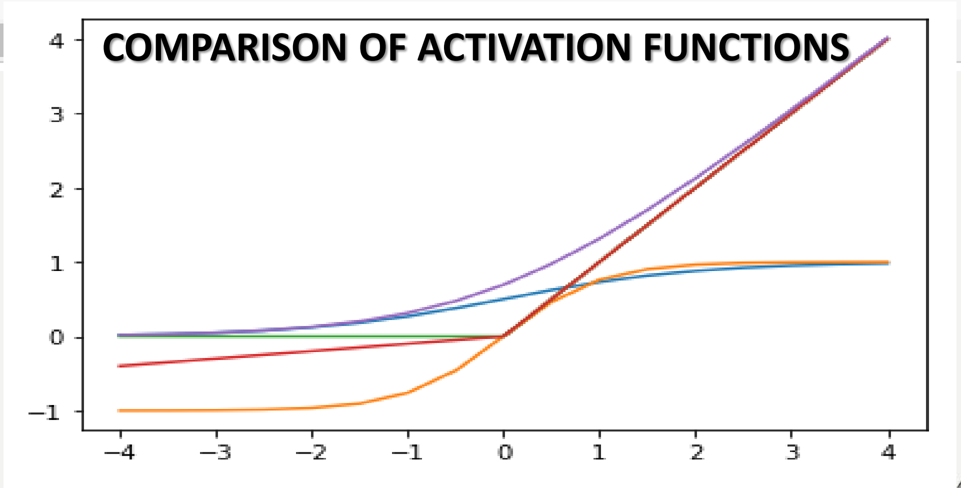
How to design the training process in parallel?

Activation Functions of Neurons



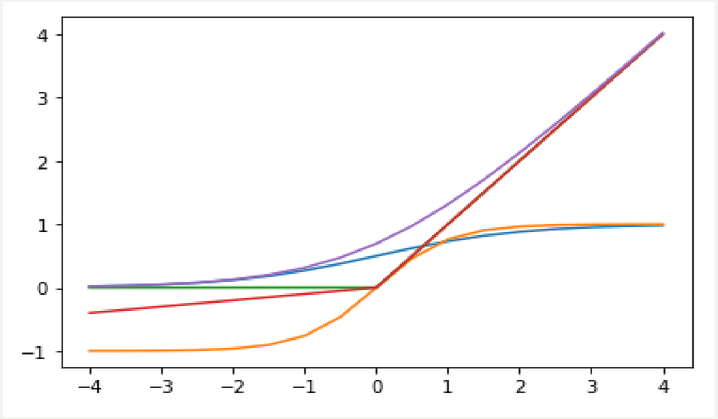
```python
import numpy as np

def sigmoid(x):
    s = 1 / (1 + np.exp(-x))   # use np.exp to implement sigmoid activation function that works on a vector or a matrix
    return s

def tanh(x):
    t = np.tanh(x)   # np.tanh to implement tanh activation function that works on a vector or a matrix
    return t

def relu(x):
    r = np.maximum(0, x)   # use np.maximum to implement relu activation function that works on a vector or a matrix
    return r

def leakyrelu(x, slope):
    l = np.maximum(x * slope, x)   # use np.maximum to implement leaky relu activation function that works on a vector or a m
    return l

def softplus(x):
    p = np.log(1 + np.exp(x))   # use np.log and np.exp to implement softplus activation function that works on a vector or a
    return p
```

We can use different activation functions of neurons in different layers of the network:



Derivatives of Activation Functions

Derivatives are necessary for the use of gradient descent:



- **Sigmoid function:**

$g(z) = \sigma(z) = \frac{1}{1+e^{-z}}$ $\quad g'(z) = \frac{dg(z)}{dz} = g(z) \cdot \left(1 - g(z)\right) = a \cdot (1-a)$

- **Tangent hyperbolic function:**

$g(z) = tanh(z) = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$ $\quad g'(z) = \frac{dg(z)}{dz} = 1 - \left(g(z)\right)^2 = 1 - a^2$

- **Rectified linear unit (ReLu):**

$g(z) = ReLu(z) = max(0, z)$ $\quad g'(z) = \frac{dg(z)}{dz} = \begin{cases} 1 & if\ z > 0 \\ 0 & if\ z \leq 0 \end{cases}$

- **Smooth ReLu (SoftPlus):**

$g(z) = SoftPlus(z) = ln(1 + e^{z})$ $\quad g'(z) = \frac{dg(z)}{dz} = \frac{e^{z}}{1+e^{z}} = \frac{1}{1+e^{-z}}$

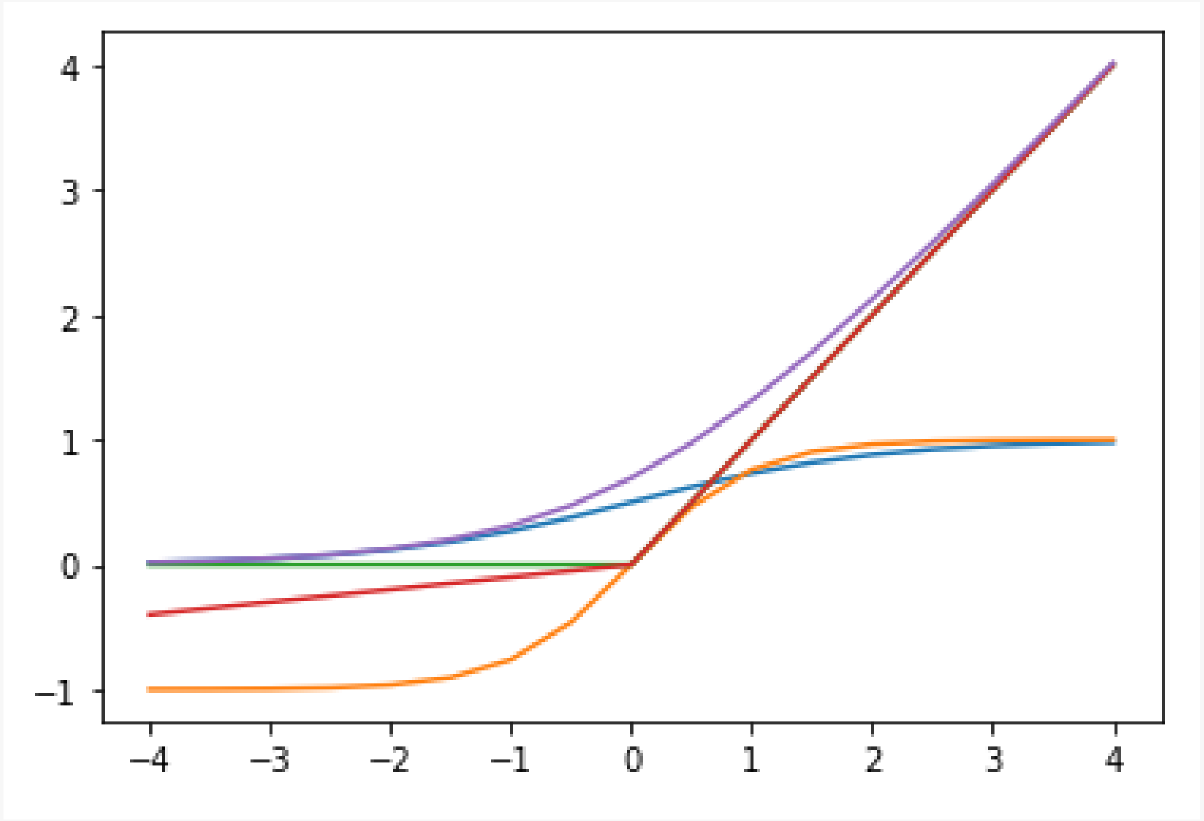
- **Leaky ReLu:**

$g(z) = LeakyReLu(z) = \begin{cases} z & if\ z > 0 \\ 0.01z & if\ z \leq 0 \end{cases}$ $\quad g'(z) = \frac{dg(z)}{dz} = \begin{cases} 1 & if\ z > 0 \\ 0.01 & if\ z \leq 0 \end{cases}$

Derivatives of Activation Functions

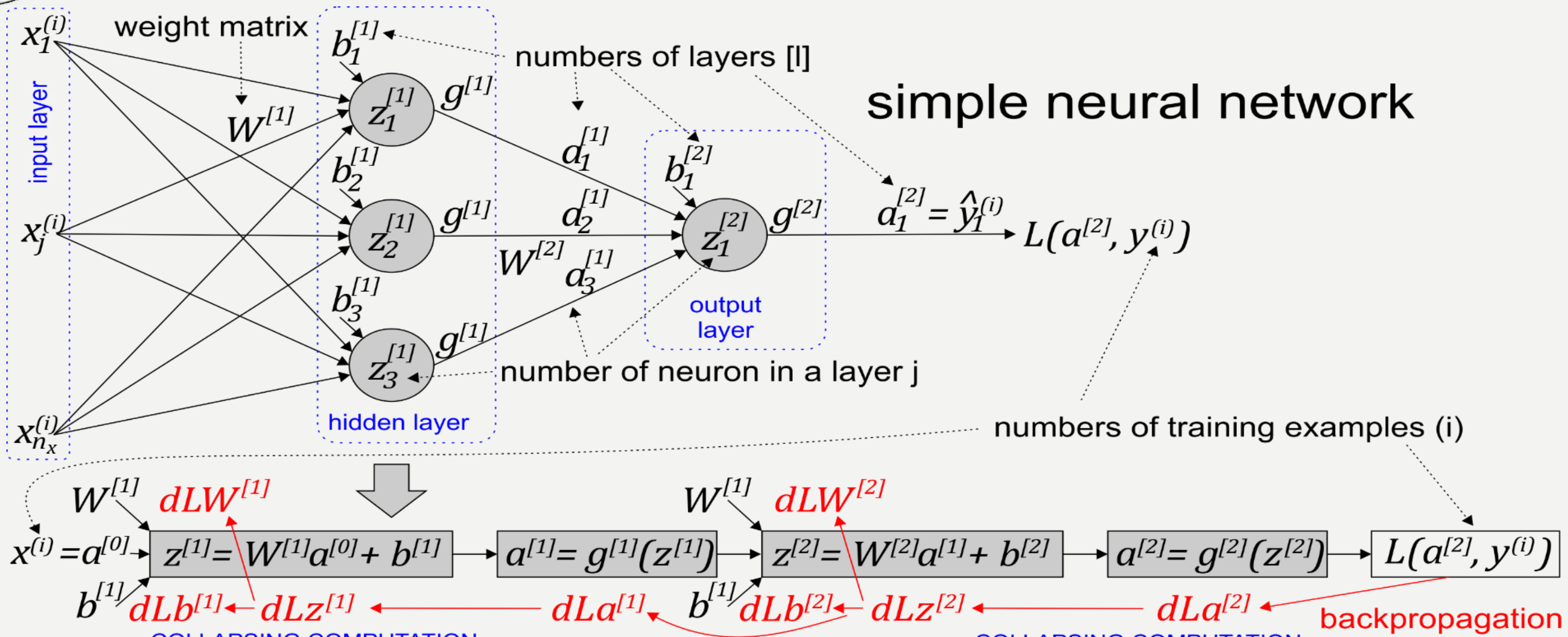
Python implementation of derivatives using numpy:

```
def sigmoid_derivative(x):
    s = sigmoid(x)
    dLs = s * (1 - s)
    return dLs

def tanh_derivative(x):
    t = tanh(x)
    dLt = 1 - t * t
    return dLt

def relu_derivative(x):
    r = relu(x)
    dLr = np.heaviside(x, 0)
    return dLr

def leakyrelu_derivative(x, slope):
    l = leakyrelu(x, slope)
    dLl = np.ones_like(x)
    dLl[x < 0] = slope
    return dLl

def softplus_derivative(x):
    p = softplus(x)
    dLp = 1 / (1 + np.exp(-x))
    return dLp
```



Neural Network Gradients

How do we propagate gradients through the network layers back?



input layer
weight matrix
numbers of layers [l]
simple neural network
$x_1^{(i)}$ $x_j^{(i)}$ $x_{n_x}^{(i)}$
$W^{[1]}$
$b_1^{[1]}$ $b_2^{[1]}$ $b_3^{[1]}$
$z_1^{[1]}$ $z_2^{[1]}$ $z_3^{[1]}$
$g^{[1]}$
$a_1^{[1]}$ $a_2^{[1]}$ $a_3^{[1]}$
$W^{[2]}$
$b_1^{[2]}$
$z_1^{[2]}$
$g^{[2]}$
$a_1^{[2]} = \hat{y}_1^{(i)}$
$L(a^{[2]}, y^{(i)})$
output layer
hidden layer
number of neuron in a layer j
numbers of training examples (i)
$x^{(i)} = a^{[0]}$
$W^{[1]}$ $dLW^{[1]}$ $b^{[1]}$
$z^{[1]} = W^{[1]}a^{[0]} + b^{[1]}$
$a^{[1]} = g^{[1]}(z^{[1]})$
$W^{[1]}$ $dLW^{[2]}$ $b^{[1]}$
$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$
$a^{[2]} = g^{[2]}(z^{[2]})$
$L(a^{[2]}, y^{(i)})$
$dLb^{[1]}$ $dLz^{[1]}$ $dLa^{[1]}$ $dLb^{[2]}$ $dLz^{[2]}$ $dLa^{[2]}$
backpropagation

COLLAPSING COMPUTATION

$dLa^{[1]} = W^{[2]T} \cdot dLz^{[2]}$

$dLz^{[1]} = dLa^{[1]} * g^{[1]\prime}(z^{[1]})$

COLLAPSING COMPUTATION

$dLa^{[2]} = -y^{(i)} \log a^{[2]} - (1-y^{(i)}) \cdot \log(1-a^{[2]}) = -y^{(i)}/a^{[2]} + (1-y^{(i)})/(1-a^{[2]})$

$dLz^{[2]} = dLa^{[2]} * g^{[2]\prime}(z^{[2]})$ * element-wise product

$dLW^{[1]} = dLz^{[1]} \cdot a^{[0]T}$ $dLW^{[2]} = dLz^{[2]} \cdot a^{[1]T}$

$dLb^{[1]} = dLz^{[1]}$ $dLb^{[2]} = dLz^{[2]}$

TRAINING EXAMPLES COLLAPSING AND VECTORIZATION

$dLZ^{[1]} = W^{[2]T} \cdot dLZ^{[2]} * g^{[1]\prime}(Z^{[1]})$ $dLZ^{[2]} = dLA^{[2]} * g^{[2]\prime}(Z^{[2]})$ * element-wise product

$dLW^{[1]} = \frac{1}{m} dLZ^{[1]} \cdot A^{[0]T}$ $dLW^{[2]} = \frac{1}{m} dLZ^{[2]} \cdot A^{[1]T}$

$dLb^{[1]} = \frac{1}{m} dLZ^{[1]}$ $dLb^{[2]} = \frac{1}{m} dLZ^{[2]}$

Random Initialization of Weights

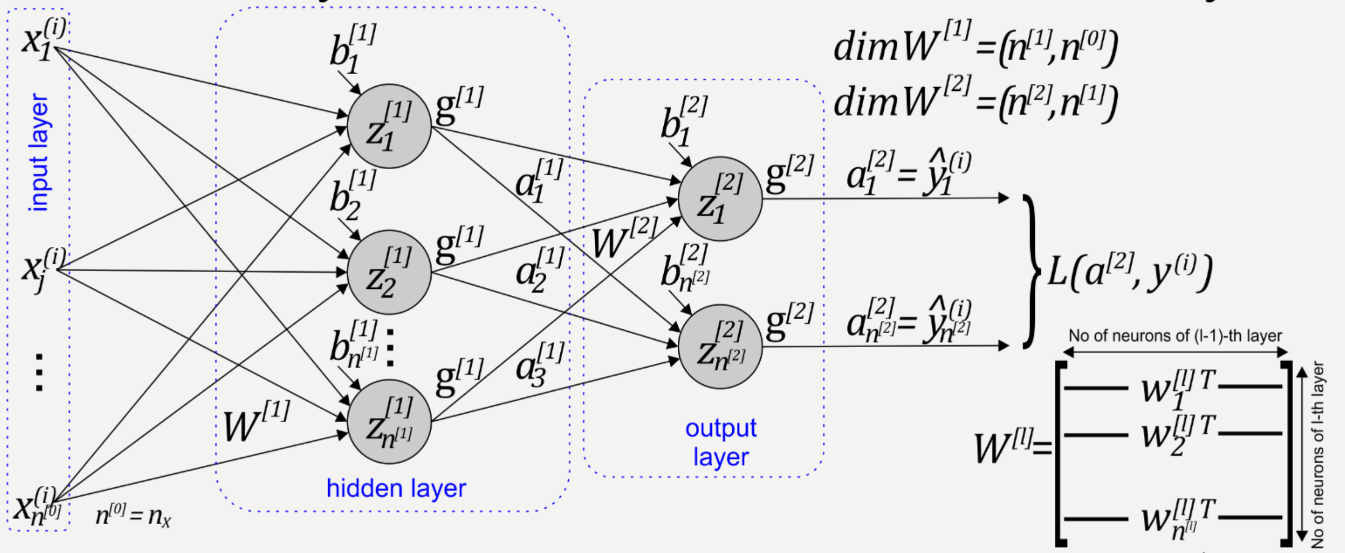
Parameters must be initialized by small random numbers, but remember that:

- **W cannot be initialized to 0:**
- $\boldsymbol{W^{[l]} = np.random.randn\left(\left(n^{[l]}, n^{[l-1]}\right)\right) * 0.01}$

- **b can be initialized to 0:**
- $\boldsymbol{b^{[l]} = np.zero\left(\left(n^{[l]}, 1\right)\right)}$

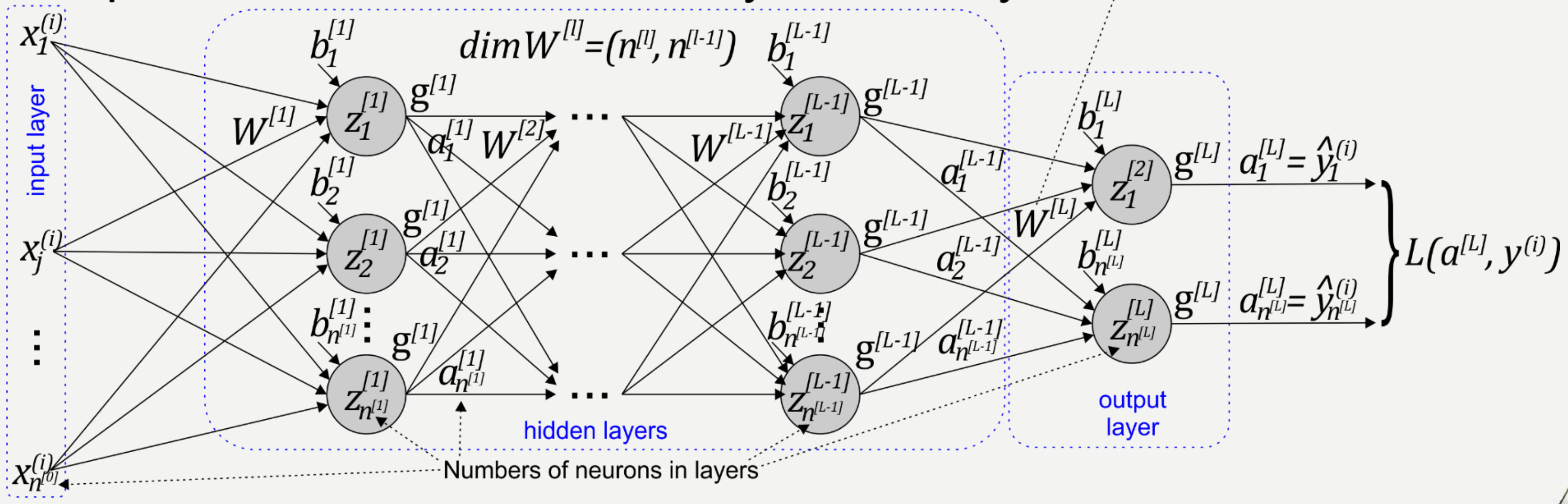
Going to Deeper NN Architectures

Shallow 2-layer NN architecture with 1 hidden layer



input layer
hidden layer
output layer
$dim W^{[1]} = (n^{[1]}, n^{[0]})$
$dim W^{[2]} = (n^{[2]}, n^{[1]})$
$a_1^{[2]} = \hat{y}_1^{(i)}$
$a_{n^{[2]}}^{[2]} = \hat{y}_{n^{[2]}}^{(i)}$
$L(a^{[2]}, y^{(i)})$
$n^{[0]} = n_x$
No of neurons of (l-1)-th layer
No of neurons of l-th layer
$W^{[l]} = \begin{bmatrix} — w_1^{[l]T} — \\ — w_2^{[l]T} — \\ — w_{n^{[l]}}^{[l]T} — \end{bmatrix}$

Deep NN architecture with many hidden layers



input layer
hidden layers
output layer
$dim W^{[l]} = (n^{[l]}, n^{[l-1]})$
$a_1^{[L]} = \hat{y}_1^{(i)}$
$a_{n^{[L]}}^{[L]} = \hat{y}_{n^{[L]}}^{(i)}$
$L(a^{[L]}, y^{(i)})$
Numbers of neurons in layers

Deep neural network architecture means the use of many hidden layers between input and output layers.

Dimensions of Stacked Matrices

$$\underset{(n^{[l]},m)}{Z^{[l]}} = \underset{(n^{[l]},n^{[l-1]})}{W^{[l]}} \cdot \underset{(n^{[l-1]},m)}{A^{[l-1]}} + \underset{(n^{[l]},1)}{b^{[l]}}$$

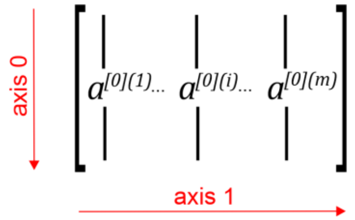
broadcasted during addition

$$\underset{(n^{[l]},m)}{A^{[l]}} = g^{[l]}\big(\underset{(n^{[l]},m)}{Z^{[l]}}\big)$$

$$\begin{bmatrix} | & & | & & | \\ z^{[l](1)} & \dots & z^{[l](i)} & \dots & z^{[l](m)} \\ | & & | & & | \end{bmatrix} = \begin{bmatrix} — & w_1^{[l]T} & — \\ — & w_2^{[l]T} & — \\ & \vdots & \\ — & w_{n^{[l]}}^{[l]T} & — \end{bmatrix} \cdot \begin{bmatrix} | & & | & & | \\ a^{[l-1](1)} & \dots & a^{[l-1](i)} & \dots & a^{[l-1](m)} \\ | & & | & & | \end{bmatrix} + \begin{bmatrix} | \\ b^{[l]} \\ | \end{bmatrix}$$

$$\begin{bmatrix} | & & | & & | \\ a^{[l](1)} & \dots & a^{[l](i)} & \dots & a^{[l](m)} \\ | & & | & & | \end{bmatrix} = g^{[l]}\left(\begin{bmatrix} | & & | & & | \\ z^{[l](1)} & \dots & z^{[l](i)} & \dots & z^{[l](m)} \\ | & & | & & | \end{bmatrix}\right)$$

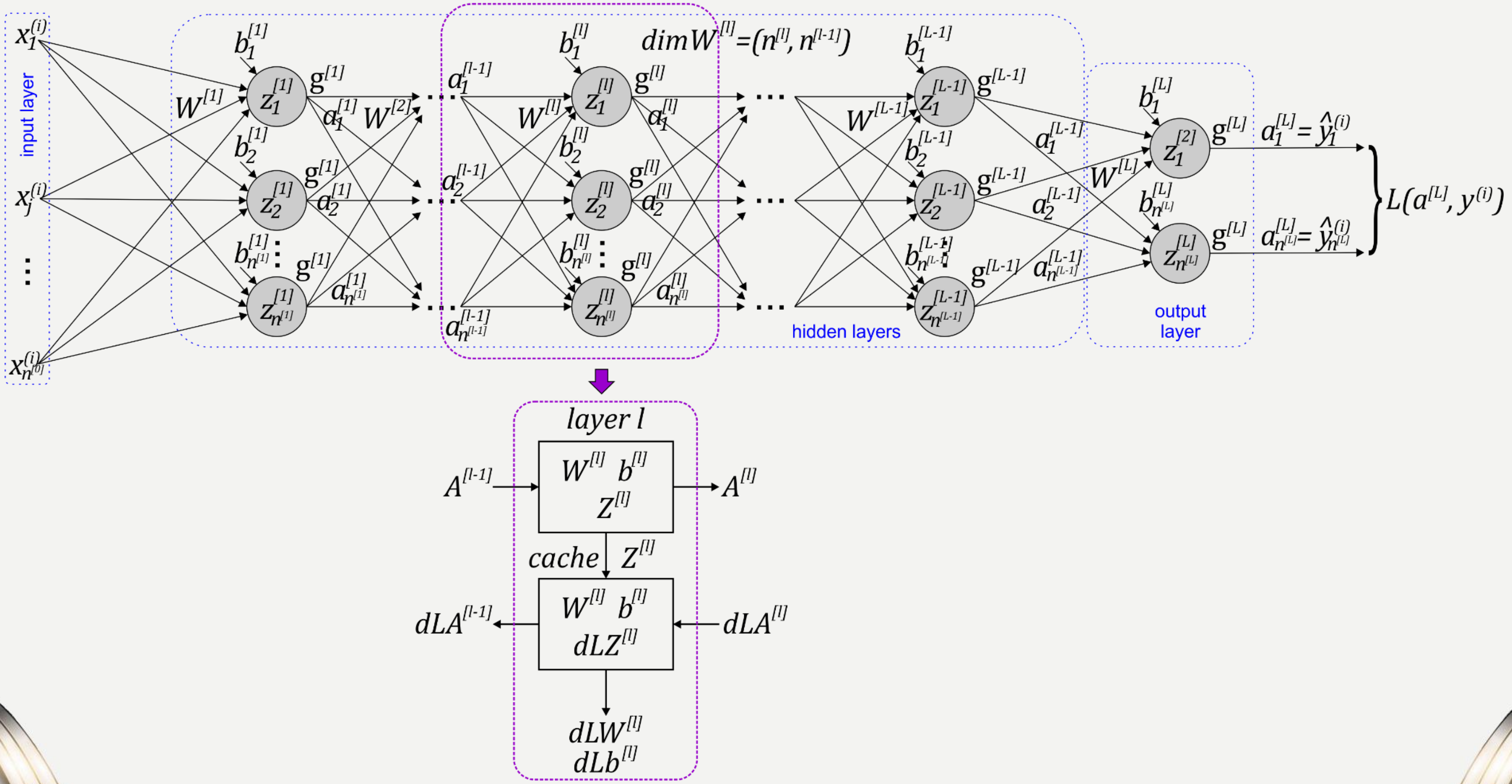
$$\begin{bmatrix} | & & | & & | \\ a^{[0](1)} & \dots & a^{[0](i)} & \dots & a^{[0](m)} \\ | & & | & & | \end{bmatrix} = \begin{bmatrix} | & & | & & | \\ x^{(1)} & \dots & x^{(i)} & \dots & x^{(m)} \\ | & & | & & | \end{bmatrix}$$



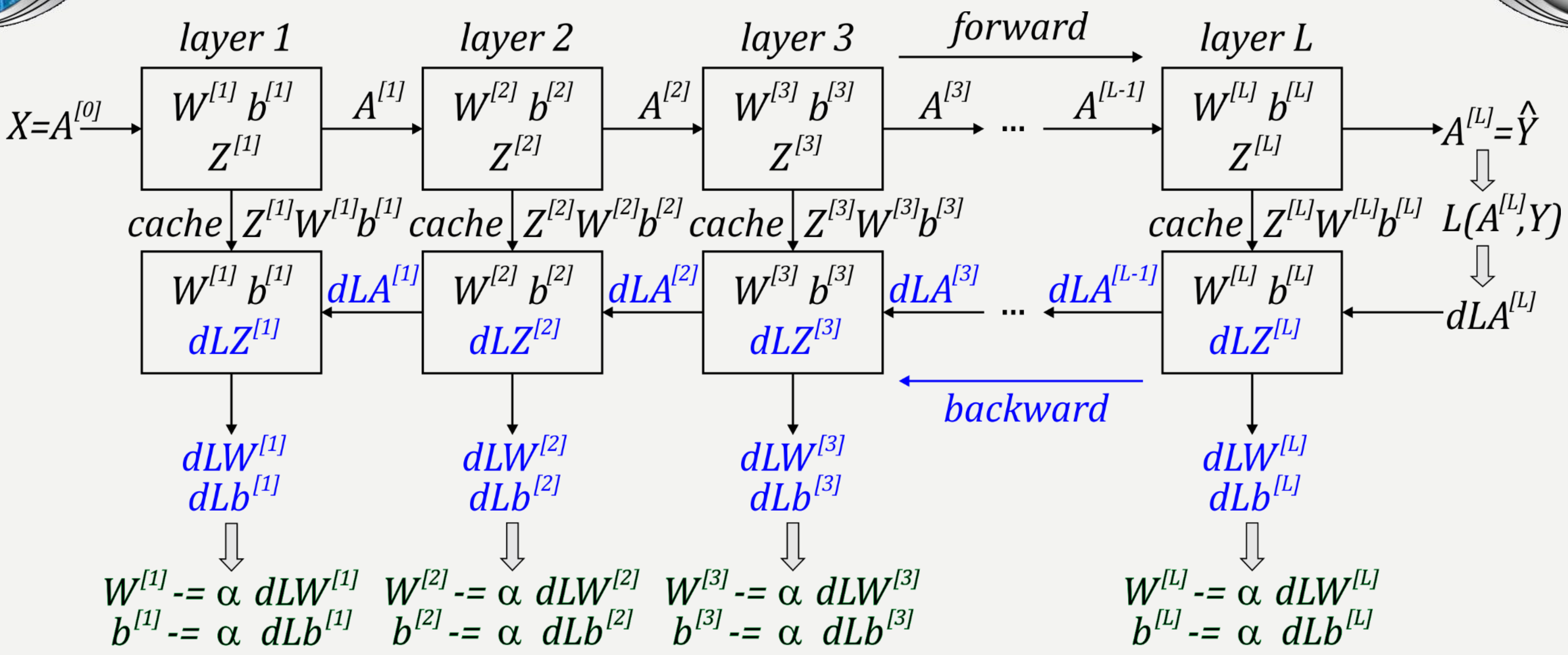
$$\underset{(n^{[l]},m)}{dLZ^{[l]}} \qquad \underset{(n^{[l]},m)}{dLA^{[l]}} \qquad \underset{(n^{[l]},n^{[l-1]})}{dLW^{[l]}} \qquad \underset{(n^{[l]},1)}{dLb^{[l]}}$$

Building Blocks of Deep Neural Networks

To design and implement the computation process using parallelism, we define blocks representing stacked neurons in layers:



Stacking Building Blocks Subsequently



layer 1
layer 2
layer 3
forward
layer L
$X=A^{[0]}$
$W^{[1]}\ b^{[1]}$ $Z^{[1]}$
$A^{[1]}$
$W^{[2]}\ b^{[2]}$ $Z^{[2]}$
$A^{[2]}$
$W^{[3]}\ b^{[3]}$ $Z^{[3]}$
$A^{[3]}$
...
$A^{[L-1]}$
$W^{[L]}\ b^{[L]}$ $Z^{[L]}$
$A^{[L]}=\hat{Y}$
cache $Z^{[1]}W^{[1]}b^{[1]}$
cache $Z^{[2]}W^{[2]}b^{[2]}$
cache $Z^{[3]}W^{[3]}b^{[3]}$
cache $Z^{[L]}W^{[L]}b^{[L]}$
$L(A^{[L]},Y)$
$W^{[1]}\ b^{[1]}$ $dLZ^{[1]}$
$dLA^{[1]}$
$W^{[2]}\ b^{[2]}$ $dLZ^{[2]}$
$dLA^{[2]}$
$W^{[3]}\ b^{[3]}$ $dLZ^{[3]}$
$dLA^{[3]}$
...
$dLA^{[L-1]}$
$W^{[L]}\ b^{[L]}$ $dLZ^{[L]}$
$dLA^{[L]}$
backward
$dLW^{[1]}$ $dLb^{[1]}$
$dLW^{[2]}$ $dLb^{[2]}$
$dLW^{[3]}$ $dLb^{[3]}$
$dLW^{[L]}$ $dLb^{[L]}$
$W^{[1]}$ -= α $dLW^{[1]}$
$b^{[1]}$ -= α $dLb^{[1]}$
$W^{[2]}$ -= α $dLW^{[2]}$
$b^{[2]}$ -= α $dLb^{[2]}$
$W^{[3]}$ -= α $dLW^{[3]}$
$b^{[3]}$ -= α $dLb^{[3]}$
$W^{[L]}$ -= α $dLW^{[L]}$
$b^{[L]}$ -= α $dLb^{[L]}$

$$dLA^{[L]} = \left(-\frac{y^{(1)}}{a^{(1)}} + \frac{1-y^{(1)}}{1-a^{(1)}}\ , \ldots,\ \frac{y^{(m)}}{a^{(m)}} + \frac{1-y^{(m)}}{1-a^{(m)}}\right)$$

$$dLA^{[l]} = W^{[l+1]T} \cdot dLZ^{[l+1]}$$

$$dLZ^{[l]} = dLA^{[l]} * g^{[l]\prime}(Z^{[l]})$$

$$\left.\right\} \quad dLZ^{[l]} = W^{[l+1]T} \cdot dLZ^{[l+1]} * g^{[l]\prime}(Z^{[l]})$$

* element-wise product

$$dLW^{[l]} = \frac{1}{m} \cdot dLZ^{[l]} \cdot A^{[l-1]T}$$

$$dLb^{[l]} = \frac{1}{m} \cdot np.sum(dLZ^{[l]}, axis=1, keepdims=True)$$

$$dLA^{[l-1]} = W^{[l]T} \cdot dLZ^{[l]}$$

Vanishing and Exploding Gradients

What are vanishing and exploding gradients and how can we deal with them?

Vanishing/Exploding Gradient Problems

In both BPTT and RTRL, we come across exploding and vanishing gradient problems:

Exploding gradients are a problem where large error gradients accumulate and result in very large updates to neural network model weights during training. This effects in instability of the model and difficulty to learn from training data, especially over long input sequences of data.

In order to robustly store past information, the dynamics of the network must exhibit attractors but, in their presence, gradients vanish going backward in time, so no learning with gradient descent is possible!

Vanishing/Exploding Gradient Problems

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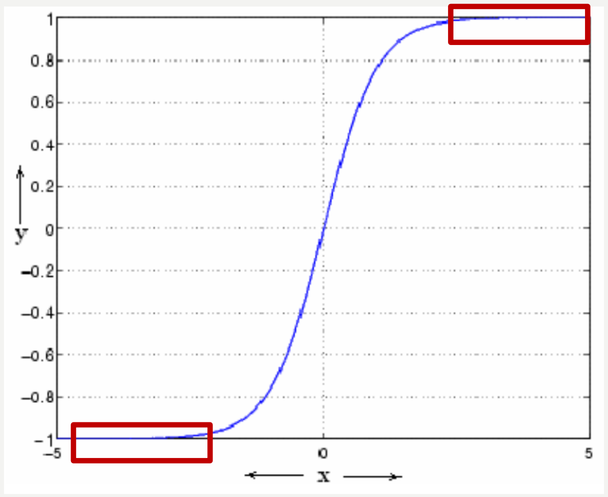
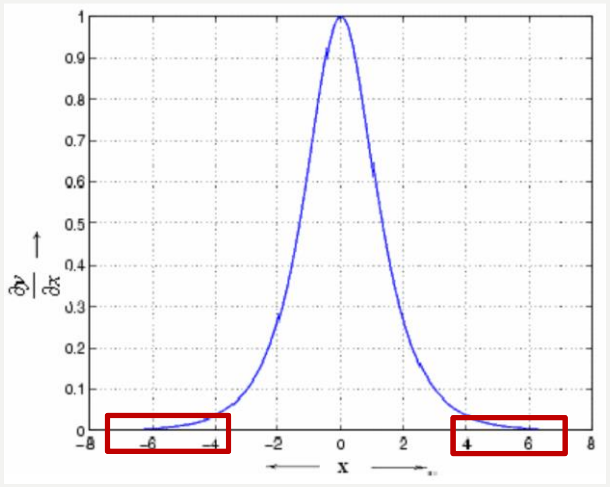




Vanishing/Exploding Gradient Problems

To reduce the vanishing/exploding gradient problems, we can:

Modify or change the architecture or the network model:

- Long Short-Term Memory (LSTM) units
- Reservoir Computing: Echo State Networks and Liquid State Machines

Modify or change the algorithm:

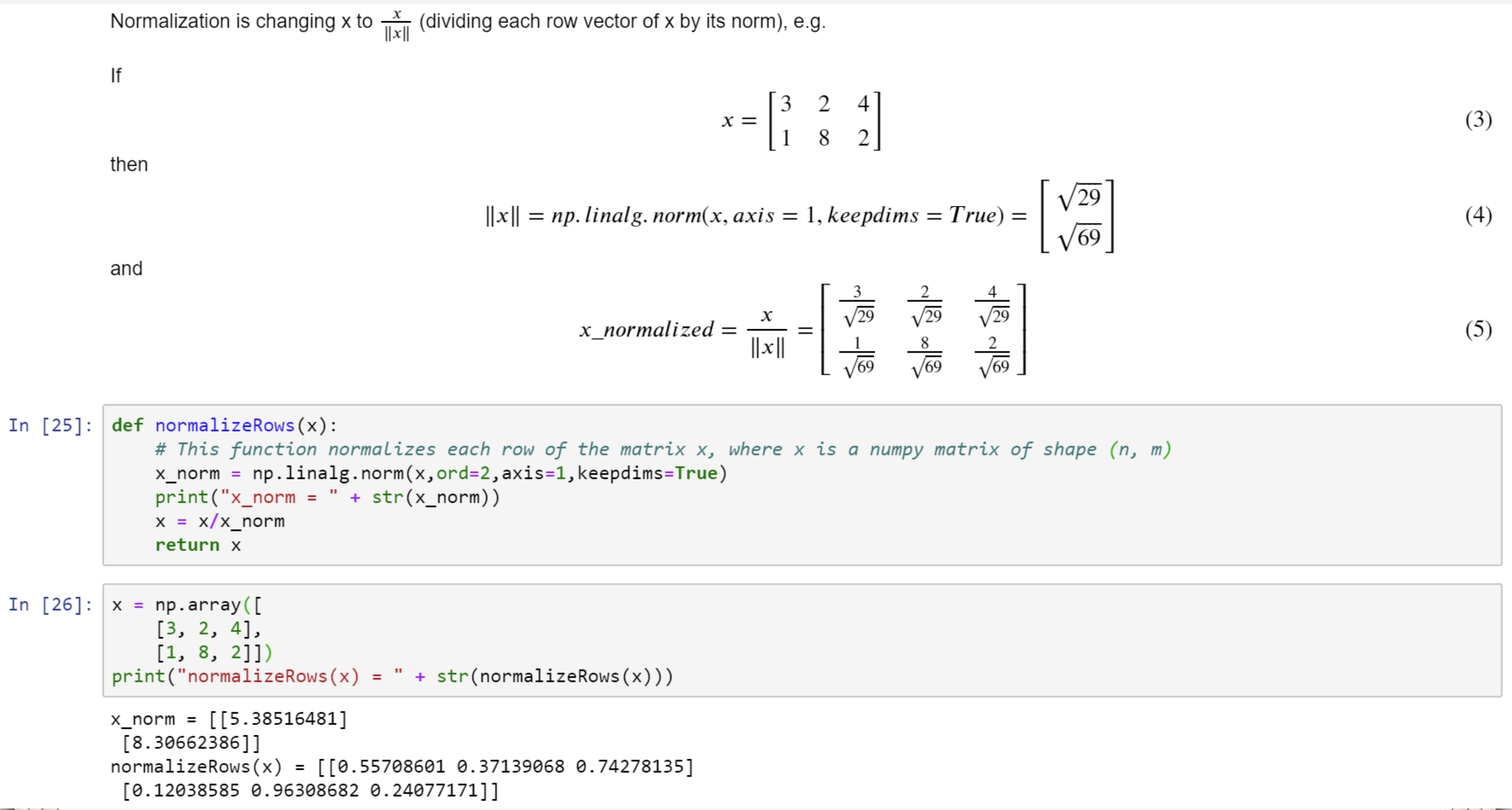
- Hessian Free Optimization
- Smart Initialization: pre-training techniques
- Clipping gradients (check for and limit the size of gradients during the training of the network)
- Truncated Backpropagation through time (updating across fewer prior time steps during training)
- Weight Regularization (apply a penalty to the networks loss function for large weight values)

Normalization

Normalization usually speeds up training.

Normalization for Efficiency

We use normalization (np.linalg.norm) to achieve a better performance because gradient descent converges faster after normalization:



Normalization is changing x to $\frac{x}{\|x\|}$ (dividing each row vector of x by its norm), e.g.

If

$$x = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 8 & 2 \end{bmatrix} \tag{3}$$

then

$$\|x\| = np.linalg.norm(x, axis = 1, keepdims = True) = \begin{bmatrix} \sqrt{29} \\ \sqrt{69} \end{bmatrix} \tag{4}$$

and

$$x\_normalized = \frac{x}{\|x\|} = \begin{bmatrix} \frac{3}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{4}{\sqrt{29}} \\ \frac{1}{\sqrt{69}} & \frac{8}{\sqrt{69}} & \frac{2}{\sqrt{69}} \end{bmatrix} \tag{5}$$

In [25]:
```python
def normalizeRows(x):
    # This function normalizes each row of the matrix x, where x is a numpy matrix of shape (n, m)
    x_norm = np.linalg.norm(x,ord=2,axis=1,keepdims=True)
    print("x_norm = " + str(x_norm))
    x = x/x_norm
    return x
```

In [26]:
```python
x = np.array([
    [3, 2, 4],
    [1, 8, 2]])
print("normalizeRows(x) = " + str(normalizeRows(x)))
```

```
x_norm = [[5.38516481]
 [8.30662386]]
normalizeRows(x) = [[0.55708601 0.37139068 0.74278135]
 [0.12038585 0.96308682 0.24077171]]
```

K-fold Cross Validation

How to validate model with the same data as are used for training it?

K-fold Cross-Validation

Cross-Validation strategy allows us to use all available examples for training and validation alternately during the training process.

„K-fold" means that we divide all examples into K disjoint more or less equinumerous subsets. Next, we train a selected model on K-1 subsets K-times and also test this model on an aside subset K-times.

The validation subset changes in the course of the next training steps:

5-FOLD	SUBSETS OF TRAINING PATTERNS				
STEPS	1	2	3	4	5
1	TEST	TRAIN	TRAIN	TRAIN	TRAIN
2	TRAIN	TEST	TRAIN	TRAIN	TRAIN
3	TRAIN	TRAIN	TEST	TRAIN	TRAIN
4	TRAIN	TRAIN	TRAIN	TEST	TRAIN
5	TRAIN	TRAIN	TRAIN	TRAIN	TEST

K-fold Cross-Validation

We use different K parameters according to the number of training patterns:

K is usually small (3 ≤ K ≤ 10) for numerous training patters.

It lets us validate the model better if it is tested on a bigger number of training patterns.

It also reduces the number of training steps that must be performed.

K is usually big (10 ≤ K ≤ N) for less numerous training datasets, where N is the total number of all training patterns.

It allows us to use more patterns for training and achieve a better-fitted model.

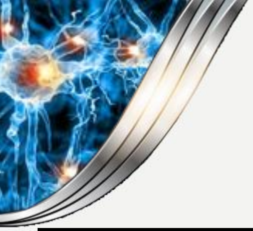
5-FOLD	SUBSETS OF TRAINING PATTERNS				
STEPS	1	2	3	4	5
1	TEST	TRAIN	TRAIN	TRAIN	TRAIN
2	TRAIN	TEST	TRAIN	TRAIN	TRAIN
3	TRAIN	TRAIN	TEST	TRAIN	TRAIN
4	TRAIN	TRAIN	TRAIN	TEST	TRAIN
5	TRAIN	TRAIN	TRAIN	TRAIN	TEST

10-FOLD	SUBSETS OF TRAINING PATTERNS									
STEPS	1	2	3	4	5	6	7	8	9	10
1	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
2	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
3	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
4	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
5	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
6	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN
7	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN
8	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN
9	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN
10	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST

K-fold Cross-Validation

N-FOLD	One-element subsets of the training patter set consisting of 20 patterns																			
STEPS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
2	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
3	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
4	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
5	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
6	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
7	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
8	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
9	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
10	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
11	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
12	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
13	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
14	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
15	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
16	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN
17	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN
18	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN
19	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN
20	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST

N-folds Cross-Validation (one-leave-out strategy) is rarely used because the N-element dataset has to be trained N times. The following disadvantage is that we use only a single pattern in each step for validation of the whole model. Such a result is not representative of the entire collection and the CI model. This solution is sometimes used for tiny datasets.



K-fold Cross-Validation

5-FOLD	SUBSETS OF TRAINING PATTERNS THAT ARE RANDOMLY ORDERED IN THE DATA SET																																							
STEPS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	TEST	TEST	TEST	TEST	TEST	TEST	TEST	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
2	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TEST	TEST	TEST	TEST	TEST	TEST	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
3	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TEST	TEST	TEST	TEST	TEST	TEST	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
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5-FOLD	SUBSETS OF TRAINING PATTERNS																																							
STEPS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN
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5-FOLD	SUBSETS OF TRAINING PATTERNS														
STEPS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN
2	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN
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4	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN
5	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST

The way of selection of the test patterns in each training step should be proportional and representative from each class point of view regardless of the cardinality of classes! We have to consider how the training data are organized in the training dataset:

- Randomly
- Grouped by categories (classes)
- Ordered by values of their attributes
- Grouped by classes and ordered by values of their attributes
- In an unknown way

K-fold Cross-Validation

5-FOLD	SUBSETS OF TRAINING PATTERNS																																							
STEPS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN
2	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST
3	TEST	TRAIN	TRAIN	TRAIN	TEST	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN
4	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN
5	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN

5-FOLD	SUBSETS OF TRAINING PATTERNS THAT ARE RANDOMLY ORDERED IN THE DATA SET																																							
STEPS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN
2	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST
3	TEST	TRAIN	TRAIN	TRAIN	TEST	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN
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5	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TEST	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TEST	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN	TRAIN

The test patterns can also be selected randomly with or without repetition.

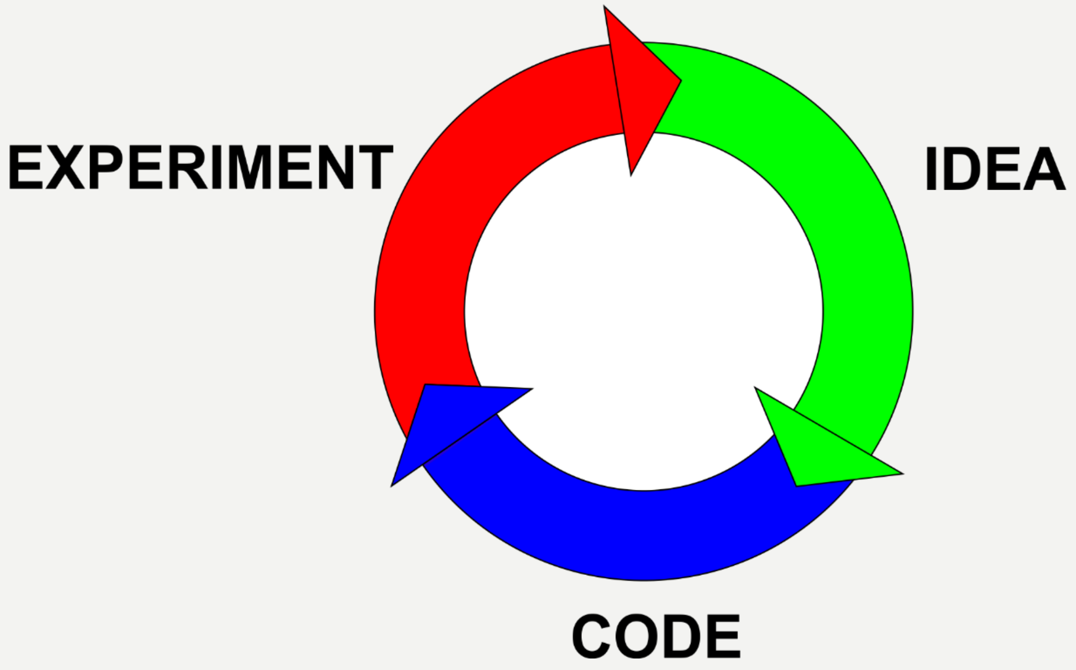
The choice between various options should be made on the basis of the initial order or disorder of patterns of all classes in the dataset to achieve representative selection of the test patterns used for the validated model.

Patterns used for validation should not be repeated in successive test groups, only that we use a less reliable and simpler approach to random choosing of validation patterns.

Optimization Process

How do we improve deep learning models?

K-fold Cross-Validation



Deep Learning solutions are usually developed in an iterative and empirical process that composes of three main steps:

- **Idea – when we suppose that a selected model, training method, and some hyperparameters let us to solve the problem.**
- **Code – when we try to code and apply the idea in a real code.**
- **Experiment – prove our suppositions and assumptions or not, and allow to update or change the idea until the experiments return satisfactory results.**

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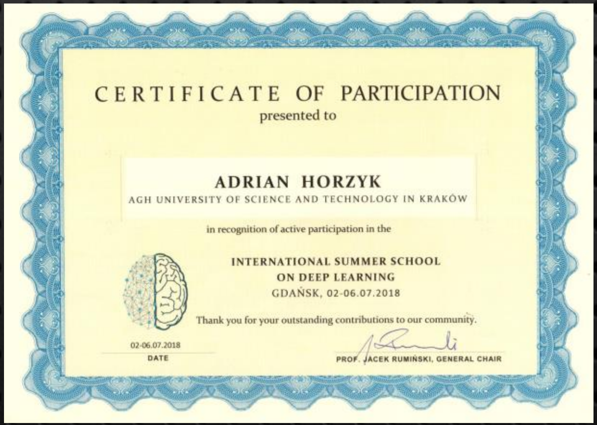

CERTIFICATE OF PARTICIPATION
presented to

ADRIAN HORZYK
AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN KRAKÓW

in recognition of active participation in the

INTERNATIONAL SUMMER SCHOOL
ON DEEP LEARNING
GDAŃSK, 02-06.07.2018

Thank you for your outstanding contributions to our community.

02-06.07.2018
DATE

[illegible]
PROF. JACEK RUMIŃSKI, GENERAL CHAIR